PROBLEM OF THE WEEK

Solution of Problem No. 8 (Spring 2003 Series)

**Problem:** Prove that there is no $2 \times 2$ matrix $S$ such that $S^r = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ for any integer $r \geq 2$.

**Solution** (by Chris Lomont, Gr. MA)

Suppose there is $S$ and $r \geq 2$ such that $S^r = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. Then $S^{2r} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. The characteristic polynomial for $S$ is $x^2 + ax + b$, hence $(x^2 + ax + b)$ is a factor of $x^{2r}$. This implies $a = b = 0$, the characteristic polynomial of $S$ is $x^2$, so $S^2 = 0$ and $S^r = S^2 S^{r-2} = 0$, thus $S^r \neq \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$.

Also solved by:

Undergraduates: Jason Andersson (So. MA)

Graduates: Tom Engelsman (ECE)

Faculty: Steven Landy (Physics at IUPUI)

Others: J.L.C. (Fishers, IN), Yalangi Chandrasekhar (Camarillo, CA), Jim Hoffman (Vincennes U.), Jeff Hammerbacher (Ft. Wayne, IN)

Four incorrect solutions were received.

We found a correct solution for problem 5 by Jason Andersson. This has been entered in the book.