PROBLEM OF THE WEEK

Solution of Problem No. 2 (Spring 2004 Series)

Problem: Show that $x = \tan 18°$ satisfies $5x^4 - 10x^2 + 1 = 0$.

Solution (by Prasenjeet Ghosh, New Delhi, India; former Purdue student)

Let $y = 18°$. Thus $5y = 90°$. Now

$$\tan(5y) = \frac{\tan(2y) + \tan(3y)}{1 - \tan(2y)\tan(3y)}. \tag{1}$$

Since $\tan 5y = \tan 90° = \infty$, it follows that

$$1 - \tan(2y)\tan(3y) = 0. \tag{2}$$

Substituting for $\tan(2y)$ and $\tan(3y)$ in Equation (2) we get

$$\left(\frac{2\tan(y)}{1-\tan^2 y}\right)\left(\frac{3\tan(y) - \tan^3 y}{1 - 3\tan^2 y}\right) = 1. \tag{3}$$

Rearranging the algebra in Equation (3), we get

$$5\tan^4 y - 10\tan^2 y + 1 = 0. \tag{4}$$

Thus $x = \tan y$ satisfies Equation (4) which is the required proof.

Also solved by:

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<u>Graduates</u>: Jianguang Guo (Phys), Ashish Rao (ECE), K. H. Sarma (Nucl)

<u>Faculty</u>: Steven Landy (Phys, IUPUI)

<u>Other</u>: Georges Ghosn (Quebec), Will Hartzell (Sr. Warren Central H.S.), Namig Mammadov (Baku, Azerbaijan), Troy Siemers (MA/CS, VMI, Lexington, VA), Christopher Smith (St. Cloud State, MN)

Two unacceptable solutions were received.