PROBLEM OF THE WEEK
Solution of Problem No. 4 (Spring 2004 Series)

Problem: Let A, B, C, D be the vertices of a tetrahedron. Prove that the altitudes from A and B meet if and only if the edges $\overline{AB}$ and $\overline{CD}$ are perpendicular.

Solution (by the Steven Landy, Fac. Physics at IUPUI)

If a line is perpendicular to a plane, it is perpendicular to any line in the plane. Thus

$$\text{altitude } \overline{AA}' \perp \overline{CD} \quad \text{and} \quad \text{altitude } \overline{BB}' \perp \overline{CD}.$$

1) If $\overline{AA}'$ meets $\overline{BB}'$, they determine a plane perpendicular to $\overline{CD}$, $\overline{AB}$ is in that plane, hence $\overline{AB} \perp \overline{CD}$.

2) If $\overline{AB} \perp \overline{CD}$ then $\overline{AA}'$ is in the plane containing $\overline{AB}$ and perpendicular to $\overline{CD}$. Similarly $\overline{BB}'$ is in that plane, hence $\overline{AA}'$ and $\overline{BB}'$ are coplanar and not parallel. Thus $\overline{AA}'$ meets $\overline{BB}'$.

Also solved by:

Undergraduates: Kedar Hippalgaonkar (Jr. ME), Paris Miles-Brenden (Jr. Phys/MA), Adam Welborn (So. CS)

Graduates: Qi Xu (Ch.E.)

Others: Georges Ghosn (Quebec), Jonathan Landy (Cal. Tech.), Angel Plaza (ULPGC, Spain)

Angel Plaza sent a late solution of Problem 3.