PROBLEM OF THE WEEK
Solution of Problem No. 5 (Spring 2004 Series)

Problem: What is the length of the day (the time between sunrise and sunset) at a place of latitude 42°N on a day when the sun's rays make an angle of 12° with the plane of the equator? (Simplifying assumptions are: the earth is a sphere, the sun's rays are parallel, and the 12° angle does not change during the day.)

Solution (by the Panel)

Let Θ° be the angle of the earth's turn, reckoned from the plane $y = 0$. The normal vector to the earth at 42° is $\underline{n} : (\cos\Theta\cos 42^\circ; \cos\Theta\sin 42^\circ, \sin\Theta)$, where we use spherical coordinates Θ for longitude, $\Phi = 42^\circ$ for latitude. The unit vector in the direction of the sun's rays is $\underline{s} : (\cos 12^\circ, 0, -\sin 12^\circ)$. Sunset happens when $\underline{n} \cdot \underline{s} = 0$, hence

$$\cos\Theta\cos 42\cos 12 - \sin 42\sin 12 = 0, \quad \text{or}$$
$$\cos\Theta = \tan 42 \cdot \tan 12.$$

There are two solutions: Θ (sunset) and $360^\circ - \Theta$ (sunrise). The length of the day in hours is $\frac{1}{15}(360 - 2\Theta) = 24 - 2\Theta/15$. Now $\Theta = \cos^{-1}(\tan 42 \cdot \tan 12) = 78.97^6$. Length of day is 13.47 hrs = 13 hrs 28 min.

Also solved by:

Undergraduates: Noah Benson (Jr. Bio/CS/MA), Akira Matsudaira (So. ECE), Paris Miles-Brenden (Jr. Phys/MA), Adam Welborn (So. CS)

Graduates: Tom Engelsman (ECE)

Faculty: Jim Dobbin (Stat), Steven Landy (Phys, IUPUI)

Others: Georges Ghosn (Quebec)

Georges Ghosn (Quebec) was incorrectly graded on Problem 1. His name should have appeared as a solver. He also points out a misprint in the published solution of Problem 3. It should be $-7 + \frac{8}{3} - 2\log 2$.