PROBLEM OF THE WEEK Solution of Problem No. 8 (Spring 2004 Series)

Problem: Masses m, 2m, $\sqrt{3}m$ are located at points P_1 , P_2 , P_3 on a circle C so that their centroid is at the center of C. Find the angles of the triangle $P_1P_2P_3$.

Solution (by the Panel)

Let C be the unit circle of the xy-plane, with mass m at the point (1,0). If $\measuredangle P_1 O P_2 = \alpha$, $\measuredangle P_1 O P_3 = \beta$, then

(1)
$$m \cdot 0 + 2m \sin \alpha + \sqrt{3} m \sin \beta = 0,$$

(2)
$$m \cdot 1 + 2m \cos \alpha + \sqrt{3} m \cos \beta = 0$$

From (1): $\sin \alpha / \sin \beta = -\frac{1}{2}\sqrt{3}$, substitute in (2) and obtain

$$1 + 2\cos\alpha + \sqrt{3}\sqrt{1 - \frac{4}{3}\sin^2\alpha} = 0,$$

whence

$$\cos \alpha = -\frac{1}{2}, \alpha = 120^{\circ},$$

so that

$$\sin\beta = -1, \quad \beta = 270^{\circ}.$$

The angles of $\triangle P_1 P_2 P_3$ are $45^\circ, 60^\circ, 75^\circ$.

Also solved by:

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One correct anonymous problem was received by fax.

Two incorrect solutions were received.