PROBLEM OF THE WEEK Solution of Problem No. 14 (Spring 2004 Series)

Problem: Show that $\cos \frac{\pi}{5} \cdot \cos \frac{2\pi}{5} \cdot \cos \frac{3\pi}{5} \cdot \cos \frac{4\pi}{5} = \frac{1}{16}$. **Solution** (by Qi Xu, Grad ChE) Let $x_k = \cos \frac{k\pi}{5}$, k = 1, 2, 3, 4.

$$x_1 = -x_4, \quad x_2 = -x_3$$

we need to prove $x_1 x_2 = \frac{1}{4}$.

From the double-angle formula

$$x_2 = 2x_1^2 - 1$$
 (1) $x_1 = -x_4 = -2x_2^2 + 1$ (2).

Hence

$$x_1 + x_2 = 2(x_1^2 - x_2^2)$$
 so $2(x_1 - x_2) = 1$.

Hence $x_2 = x_1 - \frac{1}{2}$ and from (1)

$$2x_1^2 - x_1 - \frac{1}{2} = 0.$$

Since $x_1 > 0$, $x_1 = \frac{1}{4}(1 + \sqrt{5})$ and $x_2 = x_1 - \frac{1}{2} = \frac{1}{4}(-1 + \sqrt{5})$. Thus, $x_1x_2 = \frac{1}{4}$ and $x_1x_2x_3x_4 = \frac{1}{16}$.

Also solved by:

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<u>Faculty & Staff</u>: Tim Delworth (MA), Steven Landy (Phys, IUPUI), Mark Senn (Systems Programmer)

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Three unsatisfactory solutions were received.

A correct anonymous solution was received.

A correct late solution to Prob 13 was received from Qi Xu.