## PROBLEM OF THE WEEK

Solution of Problem No. 14 (Spring 2004 Series)

Problem: Show that $\cos \frac{\pi}{5} \cdot \cos \frac{2 \pi}{5} \cdot \cos \frac{3 \pi}{5} \cdot \cos \frac{4 \pi}{5}=\frac{1}{16}$.
Solution (by Qi Xu, Grad ChE)
Let $x_{k}=\cos \frac{k \pi}{5}, \quad k=1,2,3,4$.

$$
x_{1}=-x_{4}, \quad x_{2}=-x_{3}
$$

we need to prove $x_{1} x_{2}=\frac{1}{4}$.
From the double-angle formula

$$
\begin{equation*}
x_{2}=2 x_{1}^{2}-1 \quad(1) \quad x_{1}=-x_{4}=-2 x_{2}^{2}+1 \tag{1}
\end{equation*}
$$

Hence

$$
x_{1}+x_{2}=2\left(x_{1}^{2}-x_{2}^{2}\right) \quad \text { so } \quad 2\left(x_{1}-x_{2}\right)=1
$$

Hence $x_{2}=x_{1}-\frac{1}{2}$ and from (1)

$$
2 x_{1}^{2}-x_{1}-\frac{1}{2}=0
$$

Since $x_{1}>0, x_{1}=\frac{1}{4}(1+\sqrt{5})$ and $x_{2}=x_{1}-\frac{1}{2}=\frac{1}{4}(-1+\sqrt{5})$.
Thus, $x_{1} x_{2}=\frac{1}{4}$ and $x_{1} x_{2} x_{3} x_{4}=\frac{1}{16}$.

Also solved by:
Undergraduates: Al-Sharif M.T. Al-Housseiny (Fr. ChE)
Graduates: Vikram Buddhi (MA), Tom Engelsman (ECE), Ashish Rao (ECE), K. H. Sarma (Nucl), Amit Shirsat (CS)

Faculty \& Staff: Tim Delworth (MA), Steven Landy (Phys, IUPUI), Mark Senn (Systems Programmer)
Others: Sudipta Das (Jadarpur U. India), Sumita Das (Bengal Engr Coll, Bangladesh), Georges Ghosn (Quebec), Will Hartzell (Sr. Warren Central H.S.), John R. Kolavo (Benet Acad Coll, IL), Christopher Smith (Fac., St. Cloud State, MN)

Three unsatisfactory solutions were received.
A correct anonymous solution was received.
A correct late solution to Prob 13 was received from Qi Xu.

