PROBLEM OF THE WEEK

Solution of Problem No. 1 (Spring 2005 Series)

Problem: Suppose b and c are real numbers randomly chosen in the interval [0,1]. What is the probability that the distance in the complex plane between the two roots of the equation $z^2 + bz + c = 0$ is not greater than 1?

Solution (by Georges Ghosn, Quebec, edited by the Panel)

The distance between the 2 roots, which is equal to $\sqrt{|\triangle|} = \sqrt{|b^2 - 4c|}$, is not greater than 1 if and only if $-1 \leq b^2 - 4c \leq 1$. That means that the point $M(b,c)$ lies on the intersection of the region in between the 2 parabolas $y = \frac{x^2-1}{4}$ and $y = \frac{x^2+1}{4}$ and the square delimited by $x = 0$, $x = 1$, $y = 0$, $y = 1$.

The probability is equal to the area of this region, which is $\displaystyle\int_0^1 \frac{x^2+1}{4} dx = \frac{1}{3}$, over the area of the square which is equal to 1. Consequently the probability is equal to $\dfrac{1}{3}$.

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