PROBLEM OF THE WEEK

Solution of Problem No. 13 (Spring 2006 Series)

Problem: Given is an ellipsoid K with semi–axes $a > b > c > 0$, center O. Let $P \in K$, not lying on any of the semi–axes of K. Show that there is a unique plane through O and P that cuts off an ellipse E from K such that OP is a semi–axis of E.

Solution (by Georges Ghosn, Quebec; edited by the Panel)

O is a center of symmetry for K, therefore every plane through O and P cuts off a closed curve E from K of second degree. Therefore E is an ellipse centered at O.

OP is a semi–axis of E iff OP is perpendicular to the tangent line $\triangle$ to E at P and therefore $\triangle$ must belong to the plane perpendicular to OP at P. But $\triangle$ belongs also to the plane tangent to K at P. Therefore, $\triangle$ is unique since it is the intersection of two distinct planes, which have point P in common. (It is easy to see, that they are distinct because P is not lying on any of the semi–axis of K).

Finally the two lines OP and $\triangle$ define the unique plane which satisfies the problem conditions.

Also, at least partially solved by:

Hoan Duong (San Antonio College), Steven Landy (IUPUI Physics staff)