PROBLEM OF THE WEEK
Solution of Problem No. 4 (Spring 2006 Series)

Problem: Show that

$$5x \leq 8 \sin x - \sin 2x \leq 6x$$

for $0 \leq x \leq \pi/3$.

Solution (by Georges Ghosn, Quebec; edited by the Panel)

Both inequalities can be deduced from the fact that $f(x) = 8\sin 2x - \sin 2x - 5x$ is an increasing continuous function on $\left[0, \frac{\pi}{3}\right]$ since $f'(x) = 8\cos x - 2\cos 2x - 5 = -4\cos^2 x + 8\cos x - 3 = (3 - 2\cos x)(2\cos x - 1) > 0$ on $\left(0, \frac{\pi}{3}\right)$ and $f(0) = 0$, and $g(x) = 8\sin x - \sin 2x - 6x$ is a decreasing continuous function on $\left[0, \frac{\pi}{3}\right]$ since $g'(x) = 8\cos x - 2\cos 2x - 6 = -4\cos^2 x + 8\cos x - 4 = -4(\cos x - 1)^2 < 0$ on $\left(0, \frac{\pi}{3}\right)$ and $g(0) = 0$.

At least partially solved by:

Undergraduates: Alan Bernstein (Jr. ECE), Chris Gianopoulous (Fr.), Ramul Kumar (Fr. E), Kevin Libby (So.)

Graduates: Tomek Czajka (CS), Tom Engelsman (ECE)

Others: Stephen Casey (Ireland), Mark Crawford (Sugar Grove, IL), Prithwijit De (Ireland), Steven Landy (IUPUI Physics staff), Sandeep Sarat (John Hopkins U.)