PROBLEM OF THE WEEK

Solution of Problem No. 8 (Spring 2006 Series)

Problem: The edges of a complete graph with six vertices are all colored either red or blue.

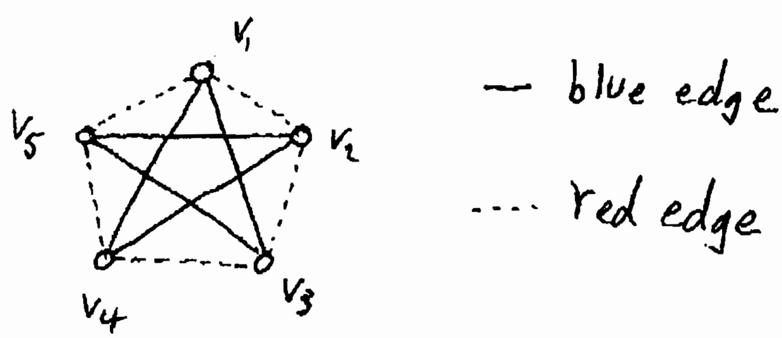
(a) Show that there exists at least one triangle with all sides of the same color.

(b) Show that (a) fails if the graph has five vertices.

Solution (by Mark Crawford, Sugar Grove, IL)

a) Let's assume we can color all the edges with either red or blue such that no triangle exists with all sides of the same color. If $\{V_1, V_2, V_3, V_4, V_5, V_6\}$ are the vertices of the graph, note that V_1 has 5 edges associated with it. Since we only have two colors, at least three edges from V_1 must be the same color. Let's call this color blue. The proof is similar if the color were red. Without loss of generality, let's say these edges are connecting with vertices V_2, V_3, and V_4. Note that if we color any edges blue between V_2, V_3, and V_4, then we will have a blue triangle with V_1 as a vertex. Therefore, the edges between V_2, V_3, and V_4 must all be red. This is a contradiction, hence there exists at least one triangle with all sides of the same color.

b) Here is an example that proves (b):



At least partially solved by:

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