PROBLEM OF THE WEEK

Solution of Problem No. 12 (Spring 2007 Series)

Problem: The harmonic series $1 - \frac{1}{2} + \frac{1}{3} - \cdots \pm \frac{1}{n} \mp \cdots$ converges to $\log 2$. Show that the rearrange series

$$1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \frac{1}{9} + \frac{1}{11} - \frac{1}{6} + \cdots$$

converges and find its limit.

Solution (by Denes Molnar, Physics, Assistant Prof.)

The k–th element of the series ($k = 1, 2, \ldots$) is

$$\begin{aligned} a_k &= 1/(4k-3) + 1/(4k-1) - 1/2k \\ &= [1/(4k-3) - 1/(4k-2) + 1/(4k-1) - 1/4k] + 1/2 \times [1/(2k-1) - 1/2k] \end{aligned}$$

The first term is the sum of TWO consecutive terms, $(2k-1)-$th and $(2k)-$th from ln[2], while the second term is HALF the k–th term from ln[2]. Summed separately, they both converge, and therefore the summed series also converges to the sum of the individual limits (elementary theorem from analysis). Thus, the rearranged series converges to $3/2 \times \ln[2]$.

Also solved by:

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