PROBLEM OF THE WEEK

Solution of Problem No. 7 (Spring 2007 Series)

Problem: Let S be a finite set of complex numbers. Prove that S contains a subset S_0 such that $\left|\sum_{z\in S_0} z\right| \geq \frac{1}{4}\sum_{z\in S}|z|$.

Solution (by the Panel)

If $z = x + iy$ then $|z| \leq |x| + |y|$, we have

$$\sum_{z\in S}|z| \leq \sum_{x+iy\in S}|x| + \sum_{x+iy\in S}|y|.$$

Set $X_+ = \{x + iy \in S : x \geq 0\}$, $\quad X_-, Y_+, Y_-$ are similarly defined. It is easy to see

$$\sum_{z\subset S}|z| \leq \sum_{z\in X_+} x + \sum_{z\in X_-}(-x) + \sum_{z\in Y_+} y + \sum_{z\in Y_-}(-y).$$

One of these 4 sums is the largest, $\geq \frac{1}{4}\sum|z|$, say $\sum\limits_{z\in Y_+} y$. Set $S_0 = \{z = x + iy \in S : y \geq 0\} = Y_+$. Then

$$\left|\sum_{z\in S_0} z\right| \geq \sum_{z\in S_0} y \geq \frac{1}{4}\sum_{z\in S}|z|.$$

Also solved by:

<u>Graduates</u>: Noah Blach, Tomek Czajka (CS)

<u>Others</u>: Georges Ghosn (Quebec), Steven Landy (IUPUI Physics), Kishin K. Sadarangani (Professor, ULPGC, Spain), Tom Sellke (Professor, Purdue)