PROBLEM OF THE WEEK
Solution of Problem No. 4 (Spring 2008 Series)

Problem: Show that $10^{2008} - 10^8$ is divisible by 2008.

Solution (by Bill Wolber Jr., ITaP/TLT/ELT SysAdmin)

$2008 = 8 \times 251$ and 251 is prime. So, by Fermat's Little Theorem ($a^p \equiv a \mod p$, whenever p is prime):

$$10^{2008} - 10^8 = 10^{8\times 251} - 10^8 = (10^8)^{251} - 10^8 \equiv (10^8 - 10^8) \mod 251 = 0 \mod 251$$

Clearly, $8|10^{2008}$ and $8|10^8$ and 8 is relatively prime to 251, so $8 \times 251 = 2008|(10^{2008} - 10^8)$.

Also solved by:

Undergraduates: Noah Blach (Fr. Math), Michael Burkhurt (Fr. Econ.), Rahul Kumar (Sr. ECE)

Graduates: Abhishek Arora (ECE), Richard Eden (Math), Jim Vaught (ECE)

Others: Aviv Adler (Jr., College Prep. HS, CA), Al-Sharif Talal Al-Housseiny (Shell Chemical, Norco, LA), Manuel Barbero (New York), Brian Bradie (Christopher Newport U. VA), Randin Divelbiss (Undergraduate, University of Wisconsin–Stevens Point), Mihaela Dobrescu (Faculty, Christopher Newport Univ.), Nathan Faber (Sr., Case Western Reserve Univ.), Elie Ghosn (Montreal, Quebec), Rob Hathaway (Engineer, Ridgeland, MS), Brian Huang (Jr. Saint Joseph's HS, IN), Patricia Johnson (OSU–Lima, OH), Matt Keti (Freshman, Univ. of California, Irvine), Gerard D. Koffi & Swami Iyer (U. Massachusetts, Boston), Pete Kornya (Faculty, Ivy Tech), Steven Landy (IUPUI Physics), Robert Myers (Bethel College), Chuck Ricks (Fort Wayne, IN), Sorin Rubinstein (TAU faculty, Israel), Steve Spindler (Chicago), Kevin Ventullo (IIT, Chicago)