PROBLEM OF THE WEEK Solution of Problem No. 9 (Spring 2008 Series)

Problem: Let a_1, a_2, \ldots, a_n be a permutation of the integers $1, 2, \ldots, n$. Call a_i "big" if $a_i > a_j$ for all j > i. (Thus a_n is automatically "big".) Find the mean number of "big" elements, where the mean is taken over all permutations of $1, 2, \ldots, n$.

Solution (by Steven Landy, IUPUI Physics staff)

In the *n*! permutations of 1, 2, ..., n, *n* is a big number every time, n-1 is a big number $\frac{1}{2}$ of the time (since it is only competing with *n*), n-2 is a big number $\frac{1}{3}$ of the time (since it only needs to be to the right of n-1 and *n*), etc. So, the total count of big number in the list is $n! + \frac{1}{2}n! + \frac{1}{3}n! + \dots + \frac{1}{n}n!$.

So the mean number is $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$.

Also solved by:

<u>Undergraduates</u>: Michael Burkhurt (Fr. Econ.), Daniel Jiang (Fr. Engr)

<u>Others</u>: Aviv Adler (Jr., College Prep. HS, CA), Manuel Barbero (New York), Brian Bradie (Christopher Newport U. VA), Mihaela Dobrescu (Faculty, Christopher Newport Univ.), Graeme McRae (Palmdale CA), Sorin Rubinstein (TAU faculty, Israel), Steve Spindler (Chicago), Peyman Tavallali (Grad. student, NTU, Singapore)

There were three additional people who gave a correct analysis but had misunderstood the statement of the problem.