PROBLEM OF THE WEEK
Solution of Problem No. 11 (Spring 2009 Series)

Problem: The set $\{3, 5, 9, 29\}$ has the property that the sum of any three of its members is a prime number. Show that there does not exist a set of five (distinct) positive integers with the same property.

Solution (by Steve Spindler, Chicago)

Assume that 5 such integers exist and consider their residues modulo 3. If any residue occurs three times, then the sum of those 3 numbers is divisible by 3, since $r+r+r \equiv 3r \equiv 0 \pmod{3}$. The only prime number divisible by 3 is 3, which is not the sum of three distinct positive integers.

Therefore, no residue occurs more than twice. But since there are 5 numbers and only 3 possible residues modulo 3, all three must occur in the list. But then the sum of these three numbers is divisible by 3 ($0 + 1 + 2 \equiv 0 \pmod{3}$), again a contradiction. Thus no such 5 integers exist.

The problem was also solved by:

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