PROBLEM OF THE WEEK
Solution of Problem No. 13 (Spring 2009 Series)

**Problem:** A homogeneous solid body (like an inverted ice cream cone) is made by joining the base of a right circular cone of height $h$ and radius $r$ to the base of a hemisphere of radius $r$. The body is placed with the hemispherical end on a horizontal table. For what value of $h/r$ will the body be in equilibrium in any position?

**Solution** (by Tairan Yuwen, Graduate student, Purdue University)

To guarantee that this body can be in equilibrium in any position, its center of gravity must be located at the center of the hemisphere.

Since this body has axial symmetry, we can build a coordinate frame shown in Figure 2 ($z$ direction is not shown), and we only need consider the $x$ coordinate of the center of gravity.

Since we want the body's center of gravity at point $O$, we can calculate it using the following formula/equation (suppose the body's density is 1)

$$\underbrace{\int_{-r}^{0} \pi(r^2 - x^2)x\,dx}_{\text{(the hemisphere's portion)}} + \underbrace{\int_{0}^{h} \pi\left(\frac{h-x}{h}\right)^2 r^2 x\,dx}_{\text{(the cone's portion)}} = 0 \quad (*)$$

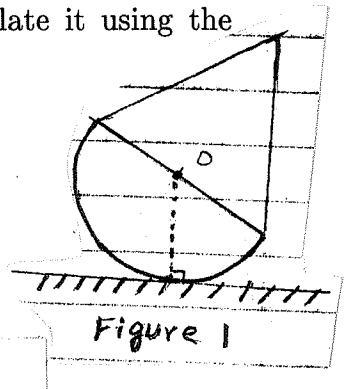

Figure 1

By simplifying (*), we get:

$$-\frac{\pi}{4}r^4 + \frac{\pi r^2 h^2}{12} = 0$$

So finally we get $\dfrac{h}{r} = \sqrt{3}$.

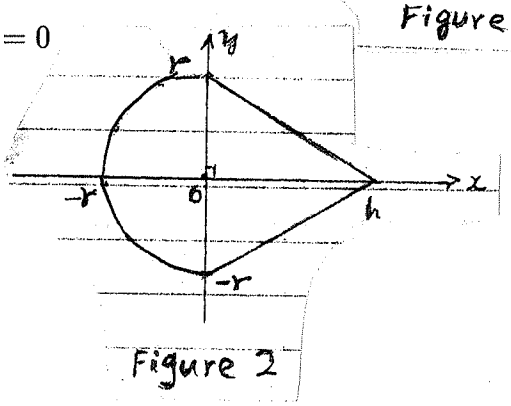

Figure 2

The problem was also solved by:

Undergraduates: Andy Bohn (Jr. Phys)

Others: Neacsu Adrian (Romania), Gruian Cornel (IT, Romania), Nathan Faber (Chemical Engineer, Parker, CO), Elie Ghosn (Montreal, Quebec), Jeffery Hein (CS & Math, Purdue Univ. Calumet), Steven Landy (IUPUI Physics staff), Kevin Laster (Indianapolis, IN), Wei-hsiang Lien (Grad student, National Chiao-Tung Univ., Taiwan), Sorin Rubinstein (TAU faculty, Israel), Craig Schroeder (Grad student, Stanford Univ.)