PROBLEM OF THE WEEK
Solution of Problem No. 2 (Spring 2009 Series)

Problem: An automobile starts from rest and ends at rest, traversing a distance of 1 mile in 1 minute along a straight road. If a governor prevents the speed of the car from exceeding 90 miles per hour, show that at some time the acceleration or deceleration of the car was at least 6.6 ft/sec^2.

Solution (by Jim Vaught, Graduate student, ECE, Purdue University)

Let $v(t)$ be the velocity in ft/s at time t seconds. Then by assumption $v(0) = v(60) = 0$ and $v(t) \leq 132\ \forall t$ on $[0, 60]$. Furthermore,

$$\int_0^{60} v(t)dt = 5280.$$

Assume by contradiction that the magnitude of the acceleration $|\frac{dv}{dt}| < 6.6\ \forall t$ on $[0, 60]$. Then on the interval $(0, 20]$, $v(t) < 6.6t$ so $\int_0^{20} v(t)dt < \int_0^{20} 6.6t\,dt = 3.3t^2\Big|_0^{20} = 1320$. Likewise on the interval $[40, 60)$, $v(t) < -6.6t + 396$ so

$$\int_{40}^{60} v(t)dt < \int_{40}^{60} (396t - 6.6t)dt = [396t - 3.3t^2]\Big|_{40}^{60} = 1320.$$

Finally, on the interval $(20, 40)$, $\quad v(t) \leq 132$ so

$$\int_{20}^{40} v(t)dt \leq \int_{20}^{40} 132dt = 132t\Big|_{20}^{40} = 2640.$$

And

$$\int_0^{60} v(t)dt = \int_0^{20} v(t)\,dt + \int_{20}^{40} v(t)\,dt + \int_{40}^{60} v(t)\,dt < 1320 + 2640 + 1320 = 5280.$$

But $\int_0^{60} v(t)\,dt < 5280$ is a contradiction. Therefore, somewhere on the interval $[0, 60]$ the magnitude of the acceleration $|\frac{dv}{dt}| \geq 6.6$.

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