PROBLEM OF THE WEEK
Solution of Problem No. 5 (Spring 2009 Series)

Problem: Let p be a polynomial with integer coefficients. If $p(0)$ and $p(1)$ are odd, show that p has no integral roots.

Solution (by Sahana Vasudevan, 7th grade, Miller Middle School, San Jose, CA)

Suppose $P(a) = 0$, where $a \in Z$. Then taking everything modulo 2, we get that either $P(1) \equiv 0 \pmod 2$, or $P(0) \equiv 0 \pmod 2$, since if $a \equiv 0 \pmod 2$, then $P(a) \equiv P(0) \pmod 2$ and if $a \equiv 1 \pmod 2$, $P(a) \equiv P(1) \pmod 2$. But $P(1) \equiv P(0) \equiv 1 \pmod 2$, and this is a contradiction. Hence, there are no integral roots of P.

The problem was also solved by:

Undergraduates: Andy Bohn (Jr. Phys), Michael Burkhart (So. Econ.), David Elden (So. Mech. Engr), Daniel Jiang (Fr. Engr), Douglas Murray (Jr. Civil Engr.), Xingyi Qin (Sr. Actuarial Sci.), Wenyu Zhang (Fr.)

Graduates: Richard Eden (Math), Huanyu Shao (CS), Phuong Thanh Tran (ECE), Jim Vaught (ECE), Tairan Yuwen

Others: Neacsu Adrian (Romania), Brian Bradie (Christopher Newport U. VA), Gruian Cornel (IT, Romania), Mark Crawford (Waubonsee Community College instructor), Ilir Dema (Toronto, ON), Randin Divelbiss (University of Wisconsin–Wausau), Erik B. Eggertsen (Jr. Oak Park and River Forest HS, IL), Tom Engelsman, Subham Ghosh (Grad student, Washington Univ. St. Louis), Elie Ghosn (Montreal, Quebec), Ali Gokal (Chicago, IL), Tigran Hakobyan (Armenia), Chun-Hao Huang (Grad student, National Central Univ. Taiwan), John Hyde (Hoover, AL), Chris Kennedy (Christopher Newport Univ.), Steven Landy (IUPUI Physics staff), Wei-hsiang Lien (Grad student, National Chiao-Tung Univ, Taiwan), Erika McGuire (Warren Central HS), José Manuel Moreno (Spain), Louis Rogliano (Corsica), Sorin Rubinstein (TAU faculty, Israel), Luis González Sánchez (Faculty, ULPGC, Spain), Craig Schroeder (Grad student, Stanford Univ.), Steve Spindler (Chicago), Peyman Tavallali (Grad. student, NTU, Singapore), Bill Wolber Jr. (ITaP), Sheng Xu (SMU), Yansong Xu (Brandon, FL)