PROBLEM OF THE WEEK
Solution of Problem No. 7 (Spring 2009 Series)

Problem: Let $a_1, \ldots, a_n$ be integers, not necessarily distinct. Show that there must be a non–empty sub–collection of $a_1, \ldots, a_n$ whose sum is divisible by n.

Solution (by Richard Eden, Graduate student, Math, Purdue University)

Let $S_k = a_1 + \cdots + a_k$, $k = 1, 2, \ldots, n$. If $S_k \equiv 0 \pmod{n}$ for some k, then $\{a_1, \ldots, a_k\}$ is our required collection of integers. So suppose now that the set of possible residues modulo n of the S_k's is $\{1, 2, \ldots, n-1\}$.
Therefore, we can find two partial sums S_i and S_j, $i > j$, with the same residue, so $S_i \equiv S_j \pmod{n}$. This means $S_i - S_j = a_{j+1} + a_{j+2} + \cdots + a_i$ is divisible by n, so our required collection is $\{a_{j+1}, a_{j+2}, \cdots, a_i\}$ which is nonempty since $i \geq j + 1$.

The problem was also solved by:

Undergraduates: Xingyi Qin (Sr. Actuarial Sci.)

Graduates: Phuong Thanh Tran (ECE), Tairan Yuwen

Others: Brian Bradie (Christopher Newport U. VA), Gruian Cornel (IT, Romania), Mark Crawford (Waubonsee Community College instructor), Randin Divelbiss (University of Wisconsin–Wausau), Erik B. Eggertsen (Jr. Oak Park and River Forest HS, IL), Tom Engelsman, Elie Ghosn (Montreal, Quebec), Tigran Hakobyan (Armenia), Chun-Hao Huang (Grad student, National Central Univ. Taiwan), John Hyde (Hoover, AL), S. Kirshanthan (St. Anthony's College, Sri Lanka), Steven Landy (IUPUI Physics staff), Timothy Lee (Rensselaer Polytechnic Institute), Wei-hsiang Lien (Grad student, National Chiao-Tung Univ., Taiwan), Sorin Rubinstein (TAU faculty, Israel), Steve Spindler (Chicago), Sahana Vasudevan (6th grade, Miller Middle School, CA), Yansong Xu (Brandon, FL)