Problem: You are to design three cubic dice, named A, B, and C. You may put any one of the numbers $1, 2, 3, 4, 5, 6$ on any face of any die. The requirement is that if all the dice are tossed, $P(A > B) > \dfrac{1}{2}$, $P(B > C) > \dfrac{1}{2}$, and $P(C > A) > \dfrac{1}{2}$, where, for example, $P(A > B)$, is the probability that the number showing on die A is greater than the number showing on die B. Show how to do it.

Solution (by John Karpis, Miami Springs, FL)

Let
$$A = \{1, 1, 5, 5, 5, 5\} \quad B = \{3, 3, 3, 4, 4, 4\} \quad C = \{2, 2, 2, 2, 6, 6\}.$$

The corresponding probabilities are

$$P(A > B) = \frac{2}{3}, \quad P(B > C) = \frac{2}{3}, \quad P(C > A) = \frac{1}{3} + \frac{2}{3} \cdot \frac{1}{3} = \frac{5}{9}$$

The problem was also solved by:

Graduates: Tairan Yuwen (Chemistry)

Others: Neacsu Adrian (Romania), Muhammad Ahsan (Pakistan), Gruian Cornel (IT, Romania), Elie Ghosn (Montreal, Quebec), Sorin Rubinstein (TAU faculty, Israel) Craig Schroeder (Grad student, Stanford Univ.), Zhengpeng Wu (Engr. Tsinghua Univ. China), Steve Zelaznik (BS. Econ. and Applied Math 06)