PROBLEM OF THE WEEK
Solution of Problem No. 2 (Spring 2010 Series)

Problem: The points in the plane are each colored blue, red, or yellow. Prove that there are two points of the same color of mutual distance unity.

Solution (by Kilian Cooley, Freshman, Purdue University)

Let point P be colored yellow. Construct a circle of radius $\sqrt{3}$ with center P. If every point on this circle is also yellow, then the proof follows as there must be a chord of unit length. Suppose, then, that there is a red point Q on this circle. Construct a circle of radius 1 centered on P and another centered on Q. It follows from trigonometry that these unit circles intersect at two points A and B, which each lie $1/2$ units in opposite directions from the segment $\overline{PQ}$ along its perpendicular bisector. Since by construction both A and B are 1 unit from P and Q, A and B must be blue (otherwise there is nothing to prove). However, A and B are separated by a unit distance. *Q.E.D.*

The problem was also solved by:

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<u>Others</u>: Neacsu Adrian (Romania), Gruian Cornel (IT, Romania), Pete Kornya (Faculty, Ivy Tech), Steven Landy (IUPUI Physics staff), Kevin Laster (Indianapolis, IN), Louis Rogliano (Corsica), Sorin Rubinstein (TAU faculty, Israel), S.Sanjiv (Palo Alto Research Center, CA), Craig Schroeder (Grad student, Stanford Univ.) Steve Spindler (Chicago)