PROBLEM OF THE WEEK
Solution of Problem No. 3 (Spring 2010 Series)

**Problem:** If $f, g$ are real–valued functions of one real variable, show that there exist numbers $x, y$ such that $0 \leq x \leq 1$, $0 \leq y \leq 1$, and
$|xy - f(x) - g(y)| \geq \frac{1}{4}$.

**Solution** (by Kevin Laster, Indianapolis, IN)

Since

$$1 = [1 - f(1) - g(1)] + [f(1) + g(0)] + [f(0) + g(1)] - [f(0) + g(0)],$$

one of the numbers

$$|1 - f(1) - g(1)|, |f(1) + g(0)|, |f(0) + g(1)|, |f(0) + g(0)| \quad \text{is at least } \frac{1}{4}.$$

Thus the relation holds for at least one of the points $(1, 1), (1, 0), (0, 1)$, or $(0, 0)$.

The problem was also solved by:

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Graduates: Rodrigo Ferraz de Andrade (Math), Gabriel Sosa (Math), Tairan Yuwen (Chemistry)

Others: Neacsu Adrian (Romania), Hongwei Chen (Christopher Newport U. VA), Gruian Cornel (IT, Romania), Tom Engelsman (Chicago, IL), Nathan Faber (CO), Elie Ghosn (Montreal, Quebec), Pete Kornya (Faculty, Ivy Tech), Steven Landy (IUPUI Physics staff), Wei-hsiang Lien (Grad student, National Chiao-Tung Univ., Taiwan), Matyas Matyas (Univ. Transilvania, Brasov, Romania), Sorin Rubinstein (TAU faculty, Israel), Craig Schroeder (Grad student, Stanford Univ.), Steve Spindler (Chicago), Thierry Zell (Ph.D, Purdue 03)