PROBLEM OF THE WEEK
Solution of Problem No. 2 (Spring 2011 Series)

Problem: Prove that an integer whose decimal representation consists of 3^n identical digits is divisible by 3^n.

Solution: (by Tairan Yuwen, Graduate student, Chemistry)

We can prove this proposition by using mathematical induction. First let's consider the integers that consist of number 1.

When $n = 1$, we have $3^1 = 3$ and the corresponding integer is $a_1 = 111$. It is obvious that $3|_{a_1}$ since $111 = 3 \times 37$.

If the proposition is ture for $n = k$, which means the integer $a_k = 111\ldots111$ (3^k digits totally) is divisible by 3^k, then the integer a_{k+1} (3^{k+1} digits totally) can be written as:

$$a_{k+1} = (1 + 10^{3_k} + 10^{2\times 3_k})_{a_k}.$$

Since the integer $1 + 10^{3_k} + 10^{2\times 3_k}$ has sum of all its digits as 3, it is divisible by 3. Since we already know that $3^k|_{a_k}$, now we have $3^{k+1}|_{a_{k+1}}$.

So the proposition is ture for integers that consist of number 1. For integers consisting of other numbers rather than 1, they are just multiples of the corresponding integers consisting of number 1, so they are divisible by 3^n as well.

The problem was also solved by:

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