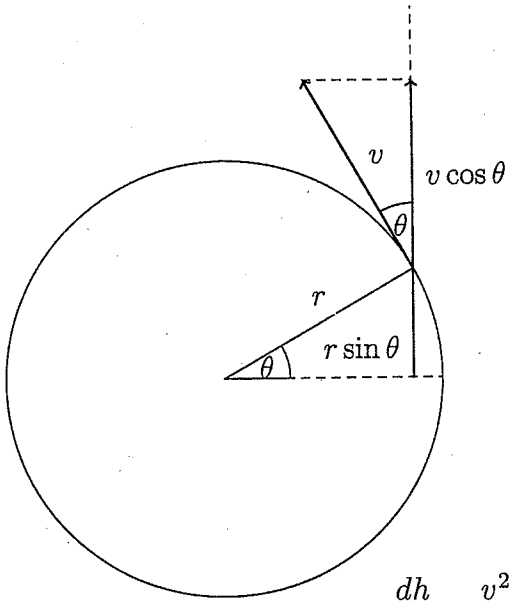
PROBLEM OF THE WEEK
Solution of Problem No. 3 (Spring 2011 Series)

Problem: A car whose wheels are of radius r feet in driven at a speed of 55 m.p.h. A particle is thrown off from one of the wheels. Neglecting air resistance, find the maximum height above the roadway which the particle can reach.

Solution: (by Shuhao Cao, Graduate student, Mathematics)

Choose the wheel as the frame of reference, suppose the particle is thrown off at a y-axis angle θ as the following figure, then the height above the road h it can reach is given by:

$$h(\theta) = \frac{v^2 \cos^2 \theta}{2g} + r \sin \theta + r, \quad 0 \le \theta \le \frac{\pi}{2}.$$



$$\frac{dh}{d\theta} = \frac{v^2 \cos \theta \sin \theta}{g} + r \cos \theta.$$

When $\frac{gr}{v^2} \le 1$, set $\frac{dh}{d\theta} = 0$ we get $\theta = \arcsin(\frac{gr}{v^2})$, the maximal height is:

$$h_{\max} = \frac{v^2}{2g} + \frac{gr^2}{2v^2} + r.$$

When $\frac{gr}{v^2} > 1$, $\frac{dh}{d\theta} > 0$ for $\forall \theta \in \left[0, \frac{\pi}{2}\right]$, hence the maximum is attained at $\theta = \frac{\pi}{2}$, the maximal height is

$$h_{\max} = 2r.$$

The problem was also solved by:

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