PROBLEM OF THE WEEK
Solution of Problem No. 1 (Spring 2012 Series)

Problem: Three hundred men sit around a circular table. The men are numbered 1–300 and each man has two neighbors. (The neighbors of 1 are 2 and 300, and the neighbors of 300 are 1 and 299.)

There are three hundred waiters, also numbered from 1 to 300. Each waiter has an urn containing three balls, one lettered L, one C and one R. Each waiter y draws a ball at random from his urn and if the ball is lettered L, delivers a dessert to the man to the left of man y. If the letter is C man y gets the dessert, and if the letter is R the man to the right of man y gets the dessert. Call a man lucky if he gets three desserts. Find the greatest possible number of lucky men, and the probability that this many men are lucky.

Solution: (by Landon Lehman, Senior, Physics, Purdue University)

It is possible for every third man to get three desserts, and so the maximum number of lucky men is $300/3 = 100$. But the condition of every third man getting three desserts can be achieved in three distinct ways: (1) the men numbered $1, 4, 7, 10, \ldots, 298$ each get three desserts, (2) the men numbered $3, 6, 9, 12, \ldots, 300$ each get three desserts. Since each of these distinct ways requires every one of the 300 waiters to do something he has a 1 out of 3 chance of doing, each way has a probability of $1/3^{300}$. But since there are three ways, the probability that 100 men are lucky is $3(1/3^{300}) = 1/3^{299}$.

The problem was also solved by:

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