PROBLEM OF THE WEEK Solution of Problem No. 11 (Spring 2012 Series)

Problem: Find a one to one area preserving map from the interior of a rectangle onto that part of the interior of the unit circle which lies above the x axis. Area preserving means the area of the image of a subregion of the rectangle is the area of the subregion, so the interior of your rectangle will have to have area $\pi/2$.

Solution 1: (by Dat Tran, Math. Graduate student, Purdue University)

Let the rectangle be $(0,1) \times \left(0,\frac{\pi}{2}\right)$. Let the map be:

$$f(r, \theta) = (\sqrt{r}\cos(2\theta), \sqrt{r}\sin(2\theta)).$$

The Jacohan of f is:

$$\begin{vmatrix} \frac{1}{2\sqrt{r}} \cos(2\theta) & \frac{1}{2\sqrt{r}} \sin(2\theta) \\ -2\sqrt{r} \sin(2\theta) & +2\sqrt{r} \cos(2\theta) \end{vmatrix} = 1.$$

So f is an area preserving continuous map from $(0,1) \times \left(0,\frac{\pi}{2}\right) \longrightarrow$ upper half of the unit disk.

The problem was also solved by:

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