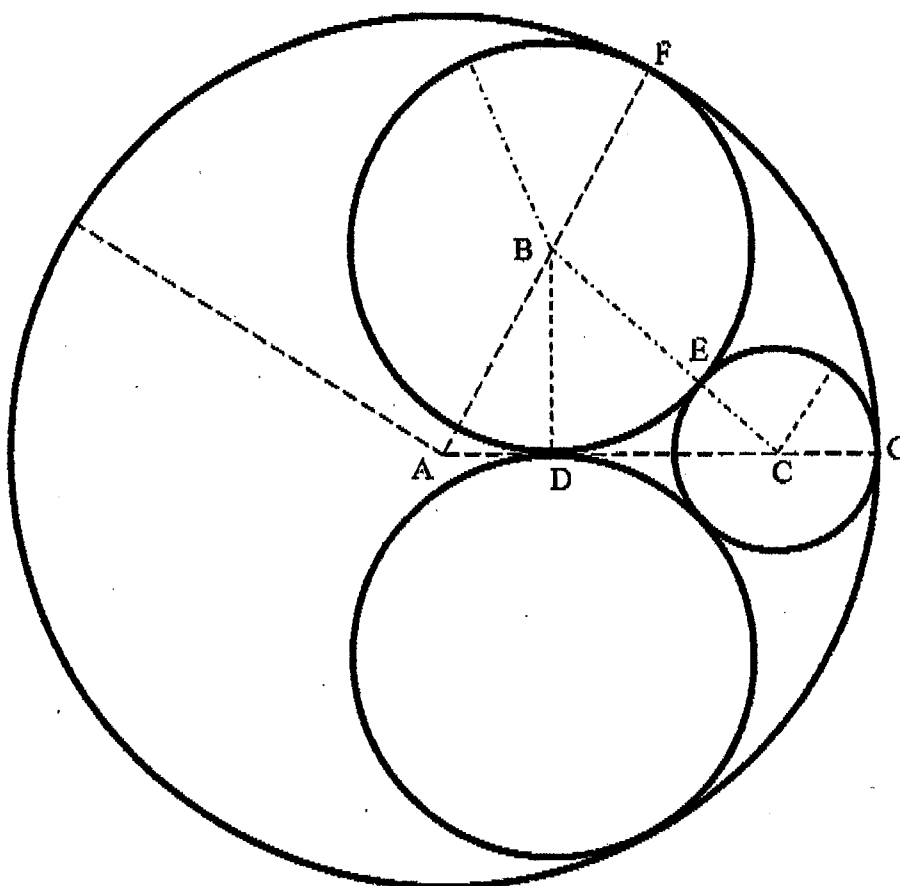


PROBLEM OF THE WEEK
Solution of Problem No. 8 (Spring 2012 Series)

Problem: Two discs of radius one and a disc of radius one half are drawn on a plane so that each of them is touching the other two at one point—think of two quarters and a penny all flat on a table and all touching at their edges. Find the radius of the largest circle which is tangent to all three of the circles which are the edges of the discs.

Solution: (by Jason L. Smith, Professor of Phys. & Math., Richland Community College, IL.)

The largest tangent circle will be drawn thus, with various radii shown.



When two circles are tangent, the radii can be arranged collinearly, as shown above in various places. The circle with center at B is one of the “quarters”, with radius $BF = BE = BD = 1$. The circle with center C is the “penny”, with radius $CE = CG = 1/2$. The largest circle with center A is the one whose radius $R = AF = AG$ is sought.

Note that triangle BCD is a right triangle with $BC = 3/2$ and $BD = 1$. Using the Pythagorean Theorem, $CD = \sqrt{5}/2$. Also note the following relationships.

$$\begin{aligned} R &= AF = AB + 1 \\ AB^2 &= AD^2 + 1 \\ R &= AG = AD + \sqrt{5}/2 + 1/2. \end{aligned}$$

This represents a system of three equations in three unknowns which can be solved for R . The solution thus obtained is $R = \sqrt{5}/2 + 1 \approx 2.11$. It may also be interesting to note that the length of DG is the golden ratio $\phi = (1 + \sqrt{5})/2$ and $R = \phi + 1/2$.

The problem was also solved by:

Undergraduates: Kaibo Gong (Sr. Math), Bennett Marsh (Fr. Engr.), Ying Xu (Fr. Engr.), Lirong Yuan (So.)

Graduates: Richard Eden (Math), Paul Farias (IE), Krishnaraj Sambath (Ch.E.), Dat Tran (Math), Tairan Yuwen (Chemistry)

Others: Manuel Barbero (New York), Pierre Castelli (Antibes, France), Pawan Singh Chawla (United Kingdom), Gruian Cornel (Cluj-Napoca, Romania), Hubert Desprez (Paris, France), Tom Engelsman (Tampa, FL), Talal Al Fares (Hasbaya, Nabatieh, Lebanon), Wilfrid Gao (HS, Valley Catholic School, OR), Elie Ghosn (Montreal, Quebec), D. Kipp Johnson (Teacher, Valley Catholic School, OR), Steven Landy (Physics Faculty, IUPUI), Joy Lin (HS, Valley Catholic School, OR), Ryan Ma (HS, Valley Catholic School, OR), Jean Pierre Mutanguha (Student, Oklahoma Christian University), Lou Poulo (Andover, MA), Brian Price (Undergrad, University of Indianapolis), Sorin Rubinstein (TAU faculty, Israel), Benjamin Tsai (NIST), Jingyu Zhang (HS, Valley Catholic School, OR), Ella Zhu (HS, Valley Catholic School, OR)