PROBLEM OF THE WEEK
Solution of Problem No. 5 (Spring 2013 Series)

Problem:
Find $\lim_{n\to\infty} \int_0^\infty \frac{e^{-x}\cos x}{\frac{1}{n}+nx^2}\,dx.$

Solution: (by Steven Landy, Physics Faculty, IUPUI)

Find $$I = \lim_{n\to\infty} \int_0^\infty \frac{e^{-x}\cos x}{\frac{1}{n}+nx^2}\,dx$$

$$I = \lim_{n\to\infty} \int_0^\infty \frac{e^{-x}\cos x}{1+n^2x^2}\,ndx = \lim_{n\to\infty} \int_0^\infty \frac{e^{-u/n}\cos(u/n)}{1+u^2}\,du.$$

Since the functions $\frac{e^{-u/n}\cos(u/n)}{1+u^2}$ are boundedly convergent for all u and n between 0 and infinity, we may use Arzela's Theorem (see Apostle, Mathematical Analysis) to move the limit inside the integral giving

$$I = \int_0^\infty \frac{1}{1+u^2}\,du = \frac{\pi}{2}.$$

The problem was also solved by:

Undergraduates: Kilian Cooley (Sr. Math & AAE), Bennett Marsh (So. Engr.), Chenkai Wang (So. Math)

Graduates: Krishnaraj Sambath (ChE), Tairan Yuwen (Chemistry)

Others: Marco Biagini (Italy), Radouan Boukharfane (Graduate student, Montreal, Canada), Charles Burnette (Grad Student, Drexel Univ.), Pierre Castelli (Antibes, France), Hongwei Chen (Professor, Christopher Newport Univ., Virginia), Hubert Desprez (Paris, France), Connor Dolan (Student, U. of New Mexico), Tom Engelsman (Tampa, FL), Ayush Gupta (Student, IIT, Delhi, India), Parviz Khalili (Faculty, Christopher Newport Univ. VA), Anastasios Kotronis (Athens, Greece), Wei-Xiang Lien (Graduate Student, National Kaohsiung Univ., Taiwan), Matthew Lim, Karthikeyan Marimuthu (Grad Student, Carnegie Mellon Univ.), Jean Pierre Mutanguha (Student, Oklahoma Christian Univ.), Christopher Nelson (Graduate Student, UCSD), Paolo Perfetti (Roma, Italy), Eric S. Proffitt, Sorin Rubinstein (TAU faculty,Tel Aviv, Israel), Craig Schroeder (Postdoc. UCLA), Mehdi Sonthonnax (Quantitative Analyst, NY), Bjorn Vermeersch (Postdoc, Purdue Univ.)