PROBLEM OF THE WEEK

Solution of Problem No. 11(Spring 2014 Series)

Problem:

Show that if f is infinitely differentiable on $(-1,1)$ and $f\left(\frac{1}{n}\right)=0$ for all $n>1$ then $f^{(k)}(0)=0$ for all $k>0$.

Solution: (by Hubert Desprez, Paris, France)

First we show by induction that for each k, there is a decreasing sequence (x_n) such that $x_n \to 0$ and for each n, $f^{(k)}(x_n)=0$, readily true for $k=0$, suppose it is true for k; the MVT says that there is an y_n, with

$$\begin{cases} 0=f^{(k)}(x_{n+1})-f^{(k)}(x_n)=(x_{n+1}-x_n)f^{(k+1)}(y_n) \\ x_{n+1}<y_n<x_n \end{cases} \quad \text{which concludes,}$$

now by continuity: $f^{(k)}(0)=f^{(k)}(\lim_{n\to\infty} y_n)=\lim_{n\to\infty} f^{(k)}(y_n)=0$.

The problem was also solved by:

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