# PROBLEM OF THE WEEK Solution of Problem No. 12(Spring 2014 Series)

## **Problem:**

How many ways are there to list 90 numbers consisting of ten ones, ten twos, ..., ten nines in a row so that for each j,  $1 \le j \le 9$ , no number bigger than j lies to the left of that j which is farthest to the left? Your answer should be in a fairly simple form.

### Solution 1: (by David Stoner, High School Student, Aiken, South Carolina)

Consider the leftmost occurrence of each digit. These need to be in the order 1,2,3,...,9, which occurs with probability  $\frac{1}{9!}$  in a given random list. There are

 $\binom{90}{(10, 10, 10, 10, 10, 10, 10, 10)} = \frac{90!}{(10!)^9} \text{ lists, so } \frac{90!}{(10!)^99!} \text{ of them are valid.}$ 

## Solution 2: (by Sorin Rubinstein, TAU Faculty, Tel Aviv, Israel)

We consider a horizontal list of 90 void entries which must be filled in. Firstly we fill in the ones. A 1 must be placed in the leftmost place in the list. The other ones may be filled in the list in  $\binom{89}{9} = \frac{89!}{9! \cdot 80!}$  ways. Then we fill in the twos. A 2 must be filled placed in the leftmost available (i.e. unoccupied) place. The other twos may be placed in the unoccupied 79 places in  $\binom{79}{9} = \frac{79!}{9! \cdot 70!}$  ways. Subsequently we fill in the threes, fours, and so on. There are:

$$\frac{89!}{9! \cdot 80!} \cdot \frac{79!}{9! \cdot 70!} \cdot \frac{69!}{9! \cdot 60!} \cdots \frac{19!}{9! \cdot 10!} \cdot \frac{9!}{9! \cdot 0!}$$

ways to fill in the whole list. Thus simplifies to:

$$\frac{89!}{(9!)^9 \cdot 10^8 \cdot 8!}.$$

### The problem was also solved by:

<u>Undergraduates</u>: Bennett Marsh (Jr. Physics & Math)

<u>Graduates</u>: Tairan Yuwen (Chemistry)

<u>Others</u>: Hubert Desprez (Paris, France), Tin Lam (Engineer, St. Louis, MO), Steven Landy (Physics Faculty, IUPUI), Esmaeil Parsa (Lecturer, Iran), Benjamin Phillabaum (Visiting Scholar, Physics, Purdue), Craig Schroeder (Postdoc. UCLA), Shin-ichiro Seki (Graduate Student, Osaka University), Christopher J. Willy (Part-time Faculty, GWU)