

PROBLEM OF THE WEEK
Solution of Problem No. 7 (Spring 2014 Series)

Problem:

Evaluate $\prod_{n=1}^{\infty} \left(1 + 10^{-2^n}\right)$.

Solution: (by Gabriel Sosa, Graduate Student, Math, Purdue University)

$$\begin{aligned}\prod_{n=1}^{\infty} \left(1 + 10^{-2^n}\right) &= \prod_{n=1}^{\infty} \frac{1 - 10^{-2^{n+1}}}{1 - 10^{-2^n}} = \lim_{k \rightarrow \infty} \prod_{n=1}^k \frac{1 - 10^{-2^{n+1}}}{1 - 10^{-2^n}} \\ &= \lim_{k \rightarrow \infty} \frac{1 - 10^{-2^2}}{1 - 10^{-2^1}} \frac{1 - 10^{-2^3}}{1 - 10^{-2^2}} \cdots \frac{1 - 10^{-2^{k+1}}}{1 - 10^{-2^k}} \\ &= \lim_{k \rightarrow \infty} \frac{1 - 10^{-2^{k+1}}}{1 - 10^{-2^1}} = \frac{1}{1 - \frac{1}{100}} \\ &= \frac{100}{99}.\end{aligned}$$

The problem was also solved by:

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