Problem:

Test for convergence the series $\displaystyle\sum_{n=1}^{\infty} a_n$**,**

where

$$a_n = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdots 2n} \quad .$$

Solution: (by Tin Lam)

The series diverges.

Note that:

$$a_n = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdots 2n} \geq \frac{1 \cdot 2 \cdot 4 \cdots (2n-2)}{2 \cdot 4 \cdots 2n} = \frac{1}{2n} = b_n.$$

Since $\displaystyle\sum_{n=1}^{\infty} b_n$ is a harmonic series, which we know diverges, by the comparison test, $\displaystyle\sum_{n=1}^{\infty} a_n$ also diverges.

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