PROBLEM OF THE WEEK Solution of Problem No. 5 (Spring 2015 Series)

Problem:

Find a polyhedron with 24 faces which has a symmetry group with 48 elements.

Solution by Bennett Marsh, Senior, Physics & Math, Purdue University

It is well-known that the symmetry group of the cube has 48 elements (24 rotations, and 24 inversion+rotations. This is "octahedral" symmetry). Therefore, if we can find a polyhedron with 24 faces and the same symmetry as a cube, that will satisfy the problems. To do this, start with a cube. On each face, place a square pyramid with equilateral triangle sides. This will create a polyhedron with $6 \times 4 = 24$ equilateral triangle faces. It also clearly has the same symmetry group as the cube. This polyhedron is known as a "tetrakis hexahedron". Another possibility is a regular octahedron with triangular pyramids placed on each face. This is a "tetrakis octahedron". A third possibility is the "deltoidal icositetrahedron", which has 24 kite-shaped faces.

The problem was also solved by:

<u>Graduates</u>: Tairan Yuwen (Chemistry)

<u>Others</u>: Hubert Desprez (Paris, France), Tom Engelsman (Tampa, FL), Matthew Lim, Sorin Rubinstein (TAU faculty, Tel Aviv, Israel), Craig Schroeder (Postdoc. UCLA), David Stoner (HS Student, Aiken, S. Carolina)