Modeling Multi-Dimensional Opinion Dynamics on Adaptive Networks

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1. Introduction: The Dynamics of Opinions

Unlike static systems, real social networks are **dynamic**: individuals influence their friends (opinion change) and may choose their friends in part based on shared views (network change). Many mathematical models simplify reality by using fixed network topologies or one-dimensional opinions (e.g, opinions on one topic).

Our Question: How do the dynamics of opinions on *multiple topics* lead to polarization or consensus?

Our Approach: We investigate the **Adaptive 2D Deffuant-Weisbuch (DW) Model** on an **Erdős–Rényi graph**. This is a graph where each edge has a fixed probability of being present or absent, independently of the other edges.

We identify four distinct convergence states that characterize the final state of the network:

1. **consensus formation** (all agree)
2. **polarization** (two dominant, opposing clusters)
3. **cluster formation** (multiple small groups)
4. **pseudo-consensus** (apparent agreement with small divisions)

Figure 1: Network analysis of partisanship

2. Methodology

We simulate agents with 2D opinions $\vec{x}_i \in [0,1]^2$ on adaptive graphs. We perform sensitivity analysis by **varying one parameter at a time** while holding others constant. These parameters are:

- β: Opinion tolerance threshold
- C: Confidence Interval
- M: Number of edges to rewire in each algorithm step
- K: Number of opinions to update in each algorithm step
- p: Probability parameter in the Erdős–Rényi graph.

Algorithm Step (t):

Rewiring (Structural): Edges where $\|\vec{x}_i - \vec{x}_j\| > \beta$ are identified. These edges are rewired with a specific probability (not 100% certainty), altering the network structure.

Interaction (Opinion): If connected agents are within confidence bound C, they converge.

$$\vec{x}_{new} = \vec{x}_{old} + \alpha \Sigma(\vec{x}_{neighbor} - \vec{x}_{old})$$

3. Current Progress (Fall 2025)

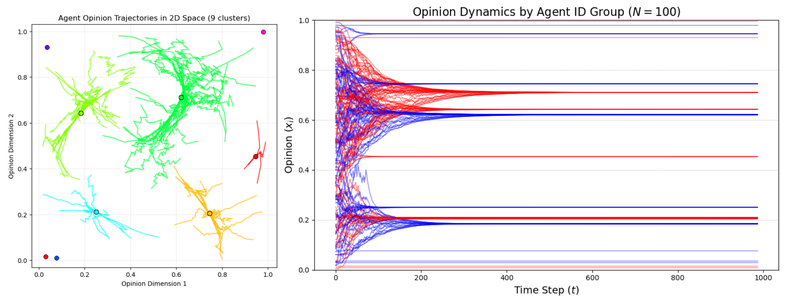

Figure 2: (a) Opinion trajectories in 2D space; (b) Cluster formation over time

We investigated the ratio of edge cuts (M) to updates (K) and identified a critical threshold.

Metastability Discovery: We observed "metastable" states where nodes settle into clusters significantly later in the simulation than in typical convergence.
Key Finding: The threshold appears around $M/K \approx 1/125$.

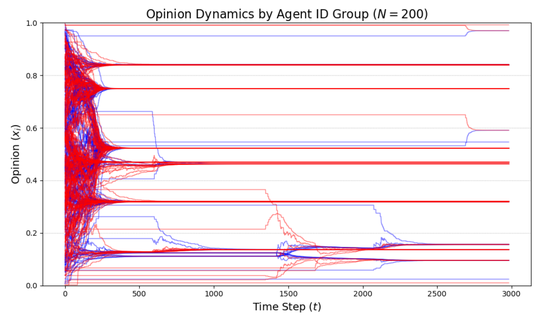

Figure 3: Opinion trajectories with cluster formation later in time.

We explored the effect of the underlying network structure on consensus formation by performing a parameter scan on p, the probability of an edge being present between two nodes.

Opinions quickly converged into a few stable clusters for small values of p. We also observed two critical thresholds of around p = 1/n and ln(n)/n.

These are closely related to the p-thresholds above which Erdos Renyi graphs contain a giant component(1/n) or are fully connected (ln(n)/n)

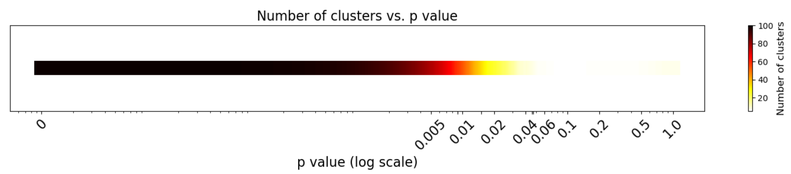

Figure 4: Parameter scan of p

We analyzed how the opinion tolerance threshold of an agent (β) and an agent's confidence interval, which is their willingness to engage with agents with different ideologies (C), affect consensus and polarization.
This is related to the previously stated rewiring step that occurs when nodes interact with one another, which determines cluster formation.
Low values of both parameters drive polarization, while high values promote consensus.

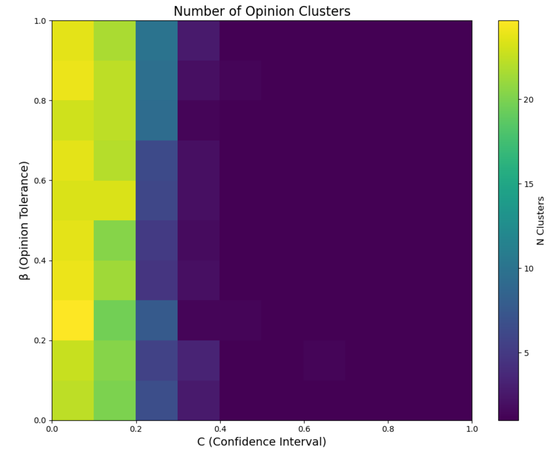

Figure 5: Parameter scan of β vs. C

4. Future Work: (Spring 2026)

1. **Theoretical thresholds:** Derive why $M/K \approx 1/125$ is the tipping point. Does it depend on β or C?
2. **Topological analysis of "late changers":** Analyze nodes that switch clusters late in the simulation (metastability). Are these high-degree "hubs" or peripheral nodes?
3. **Complex graph structure:** Explore other graph models to find relationship between network structure and consensus formation.
4. **Probability of consensus or polarization:** Analyze other factors that drive reduction in the number of clusters, making consensus more or less probable.
5. **Data:** Infer parameter values from real social networks.

5. Acknowledgements

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- **Dr. Alexandria Volkening**

6. References

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2. Wolfram."Network Analysis of Partisanship." Christopher Wolfram, 2022