Characterizing Visibility Graphs of Polygons

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Abstract

When studying the visibility graph characterization problem, we were not able to find a tool to easily compute the interior visibility graph of a polygon. Using geometric region approximation methods, we developed an interactive web application to quickly compute the visibility graph of small polygons (ten or fewer vertices). This works for quickly sketching out visibility graphs and could be used to find patterns, potentially leading to new discoveries in this field. This could eventually be scaled up by streamlining computation with faster algorithms and improved graphics.

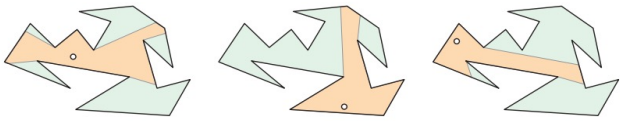
Introduction

Computational Geometry is a field at the intersection of Math and Computer Science which studies algorithms which can be used to solve problems in discrete geometry, especially in low dimension. The field has applications in robotics, computer graphics, and geographic information systems (GIS).

The Art Gallery Problem

One of the fundamental problems in Discrete Geometry is the Art Gallery Problem. We begin by modeling an art gallery as a polygonal region. The key question to be answered is: **How many guards are needed to ensure that every point inside the art gallery is visible to at least one guard?**

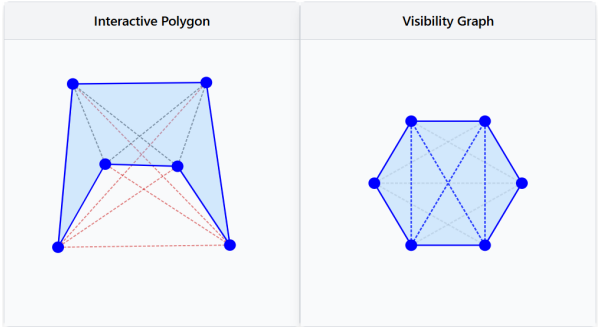
- Basic Art Gallery Theorem is known: $\lfloor \frac{n}{3} \rfloor$ guards suffice for a simple polygon with n vertices. (Via Triangulation) (1)
- Many Variations of the problems exist: what if the walls were transparent, what if the walls were mirror, and many more
- Below is an art gallery with 3 distinct guard placements (1)

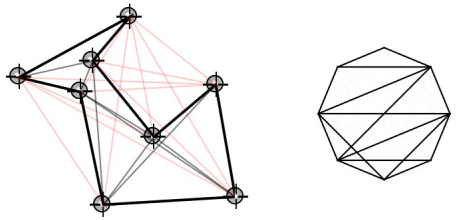


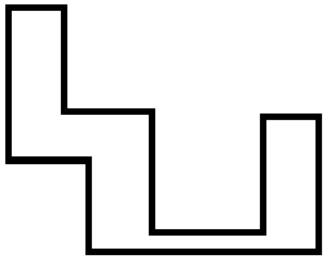
The Visibility Graph Problem

The Visibility Graph of a polygon is a graph representation of which vertices can "see" each other.

- Bijection between graph nodes and polygon vertices. There exists an edge between two nodes if their corresponding vertices can see each other.
- Unsolved Problem: **Deciding whether a given graph is the visibility graph of some polygon, and furthermore which sort of polygon it is.**
- At least three are fully known: funnels, spirals, and staircases.
- We made some progress on the characterization of rectangular pipes.







Rectangular Pipes

We found some weak results for the visibility graphs of rectangular pipes, based on the results of the Abello et al paper on visibility graphs of staircase polygons (3).

We also proved that there is an injection from the set of visibility graphs of polygons to the set of visibility graphs of polygons with integer coordinate vertices (though it is possible that this theorem was already known). This means that this restriction does not simplify the problem.

Visibility Graph Visualization Tool

During our research to classify visibility graphs, we identified a gap: there was no existing tool to manipulate a polygon and immediately visualize its visibility graph. To address this, we developed a tool that allows users to manipulate the vertices of a polygon and automatically see a generated visibility graph of that polygon. This tool will be key in future conjecture development for visibility graphs.

Mentored by Dr. Shuyi Weng and Leo Shen

References

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3. Abello, Egecioglu, and Kumar. Visibility Graphs of Staircase Polygons and the Weak Bruhat Order, I: from Visibility Graphs to Maximal Chains (1995)

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