Stochastic solutions of the quantum Yang-Baxter equation

Purdue REU

Supervisor: Keshav Dahiya

Supervisor email: kkeshav@purdue.edu

Motivation. Stochastic R-matrices encode local transition probabilities while satisfying the quantum Yang-Baxter equation, an identity underlying exactly solvable lattice models. Kuniba, Mangazeev, Maruyama, and Okado [1] constructed such matrices from symmetric tensor representations of $U_q(A_n^{(1)})$. This project investigates analogous constructions for fundamental representations of quantum loop algebras of classical types, connecting linear algebra, probability, and algebraic combinatorics.

Research objectives.

(1) **Type A: exterior powers.** Study the fundamental $U_q(L\mathfrak{sl}_{n+1})$-modules, whose finite-type restrictions are $V_r = \bigwedge_q^r \mathbb{C}^{n+1}$, $1 \leq r \leq n$. For the R-matrices on $V_r \otimes V_s$, seek compatible gauges and normalizations preserving the quantum Yang-Baxter equation and satisfying the column sum equal to 1 on explicit parameter domains. Antisymmetric fusion [2] supplies a starting point; stochasticity must be established separately.

(2) **Types B, C, and D.** Extend the investigation toward all fundamental modules of $U_q(L\mathfrak{g})$ in these types, beginning with low-rank examples and including the spin representations in types B and D. Seek general constructions and identify obstructions within specified classes of gauges.

(3) **Vertex models and symmetric functions (time permitting).** Use successful constructions as weights of stochastic vertex models. For suitable fixed boundary conditions, seek explicit partition-function formulas and investigate symmetry, branching identities, and specializations. Compare the resulting functions with deformations of Schur functions, Hall-Littlewood functions, and classical-group characters.

Undergraduate approach and outcomes. Students will learn the necessary representation theory through small matrices and combinatorial bases, using SageMath/Mathematica/SymPy to solve intertwining equations, explore gauges, and formulate conjectures. The initial milestone is a rigorously verified low-rank type-A construction or an obstruction for a stated gauge class; broader families are subsequent goals. Deliverables include documented computations, proofs in tractable cases, and a final report.

Prerequisites. Linear algebra, elementary probability, and basic programming are required; abstract algebra is helpful.

References

[1] A. Kuniba, V. V. Mangazeev, S. Maruyama, and M. Okado, *Stochastic R matrix for $U_q(A_n^{(1)})$*, Nucl. Phys. B **913** (2016), 248–277. arXiv:1604.08304.

[2] A. Ayyer and A. Kuniba, *Multispecies inhomogeneous t-PushTASEP from antisymmetric fusion*. arXiv:2503.00829.

If you are interested in participating in this research project, please send an email to the supervisor email listed above.