5) If F is a global field we again have
a) $D=F$
b) $D=F'$ is a separable quadratic ext. of F
c) $D=\mathbb{H}$ is the quaternions on F', canonical involution given by inner automorphism, $F'=F$.

(specific) d) If F' is a separable quadratic extension of F & D_0 is a set of quaternions w/center F, then $D=D_0 \otimes_F F'$ has an of the 2^{nd} kind, the tensor product of the canonical involution of D_0 on F & σ

Let ρ be a place of F' & $d_\rho \in \mathbb{Q}/\mathbb{Z}$ be the local invariant w.r.t. ρ on the left module D. We have $d_\rho = 0$ (that is, $D_\rho = D \otimes_{F'} F'_\rho$ is an algebra of matrices) for almost all ρ & $\sum_\rho d_\rho = 0$. Up to iso., D is characterized by these local invariants.
(Consequence of product formula)

(general) d) F' is a quadratic, separable ext. of F, w/nontrivial automorphism σ. Let D be a left module with centre $F' \ni \begin{cases} d_\rho = 0 \text{ if } \rho = \rho^\sigma \\ d_\rho + d_{\rho^\sigma} = 0 \text{ o.w.} \end{cases}$

Then, D admits an involution extending σ.

These are necessary conditions b/c $d_\rho((D^\sigma)^o) = -d_{\rho^\sigma}$, by def. of 2^{nd} (& remark) type & §4
Inversely, these conditions imply $D=(D^\sigma)^o$ & $\alpha \in F$ (see remark after def'n of 2^{nd} type) is a local norm $\forall \rho$ & hence is the norm of an element of F'.

This completes the list of involutions

§5 Orthogonal Sums

Def: If W & W' are ε-Hermitian right D-modules, the direct sum $W'' = W \dotplus W'$ is a right D-module with a unique ε-Hermitian product $\ni$ W & W' are orthogonal & this extend the ε-Hermitian products on W & W'. We call this the orthogonal sum of W & W' & denote it by $W \oplus W'$.

Def: A degenerate ε-Hermitian space is an orthogonal sum $W \oplus V$ of (nondegenerate$^{\varepsilon \neq 0}$) ε-Hermitian space W & a space V which has its product being 0.

Def: By convention, space W with null Hermitian product is said to be of type 2.

Type 2 spaces are finite dim'l D-modules whose unitary grp is the grp of isomorphisms $GL_D(W)$.

This def'n is for convenience. It allows uniform results on unitary & linear grp's.

Def: A nondegenerate ε-Hermitian space is said to be of type 1.

Def: The orthogonal sum is compatible w/ isometries: it puts an Abelian semigroup structure on the set of isometric classes of ε-Hermitian spaces on D. This is called the Witt-Grothendieck semigroup. The group constructed from this semi-group is called the Witt-Grothendieck group.

Def: Let H be an ε-Hermitian hyperbolic plane on D. The quotient of the Witt-Grothendieck grp w/ $\langle H \rangle$ ($\langle H \rangle \cong \mathbb{Z}$ is the grp. gen. by isometry classes of H) is called the Witt group.