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Multi-Agent Consensus Equilibrium for Range Compressed Holographic Surface Reconstruction

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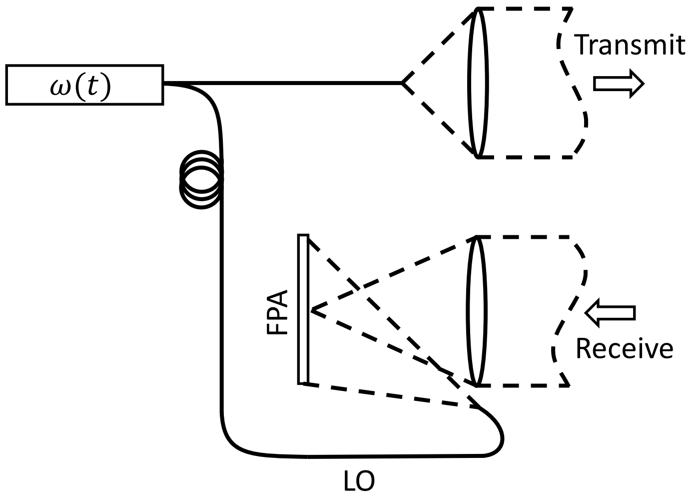
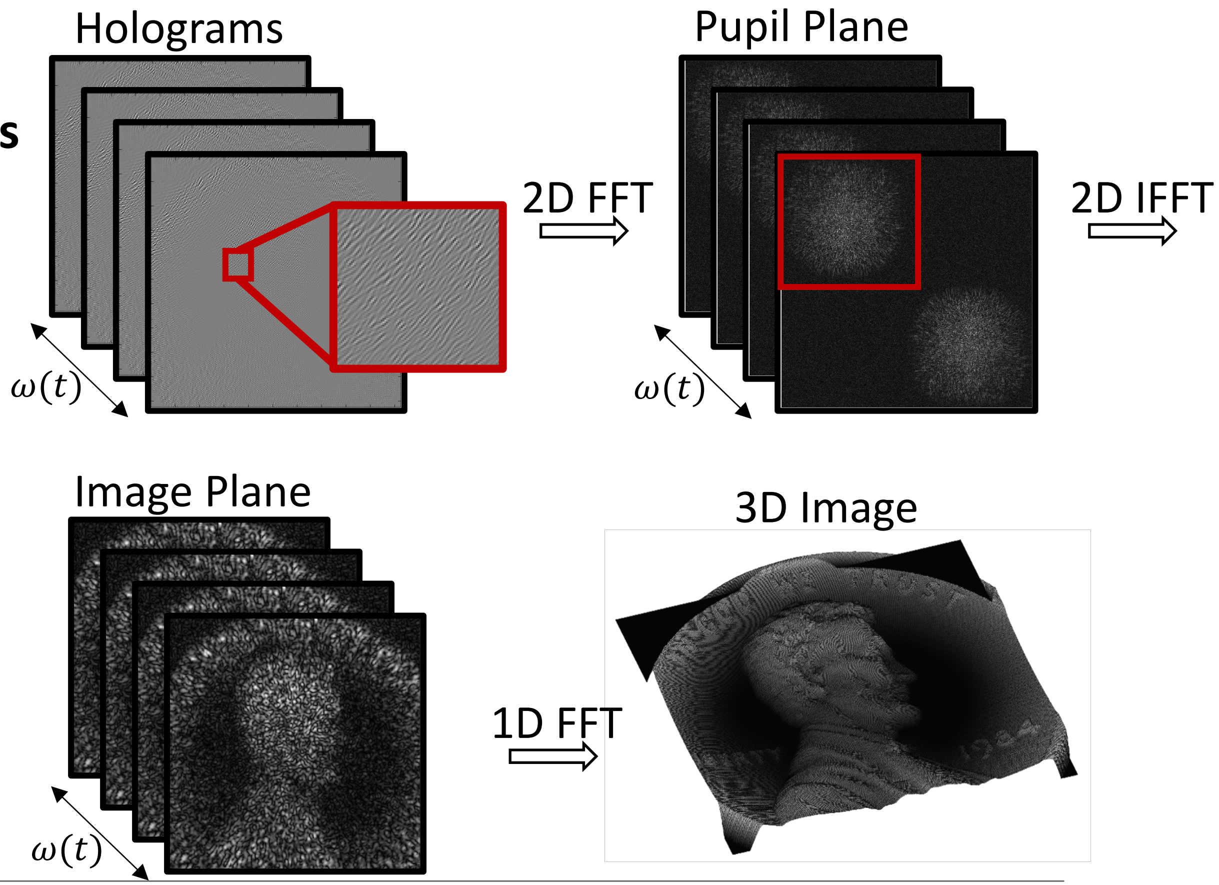
Range Compressed Holography Background

- **Form Holograms by interfering signal with off-axis reference**
  - Strong reference beam enables shot-noise limited detection
- **Use frequency diversity to increase range resolution**
  - $\Delta z = c/2B$ and $z_{amb} = c/2\Delta\omega$
- **Numerically process & filter to obtain complex images**
  - Access to phase allows for aberration correction
  - Current methods limited in low SNR regimes





RCH Model

- Last slide summarized as a linear equation

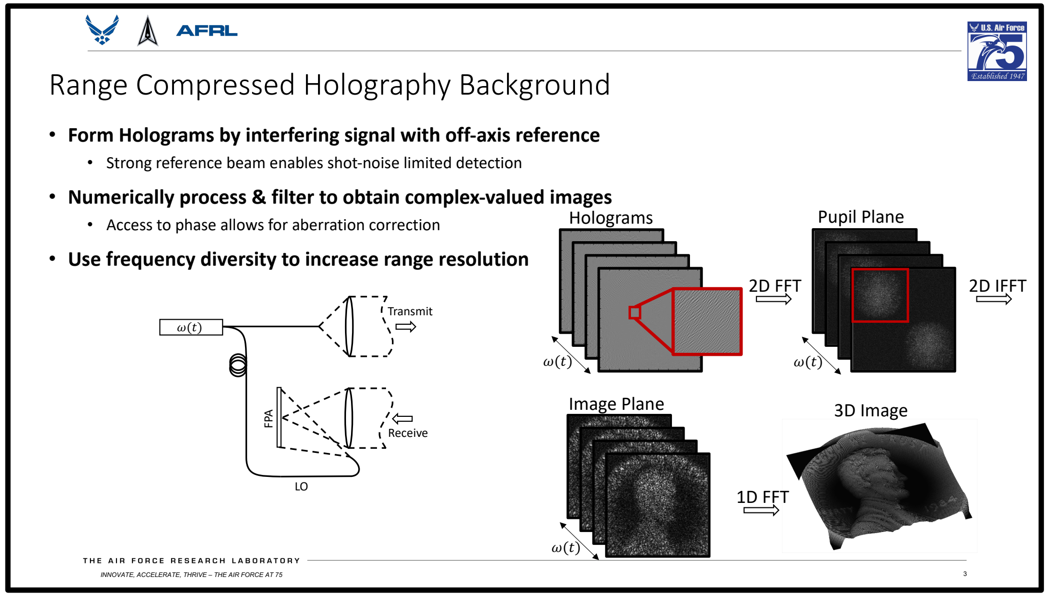
$$y = Ag + w$$

$y \in \mathbb{C}^N$ - measured complex field

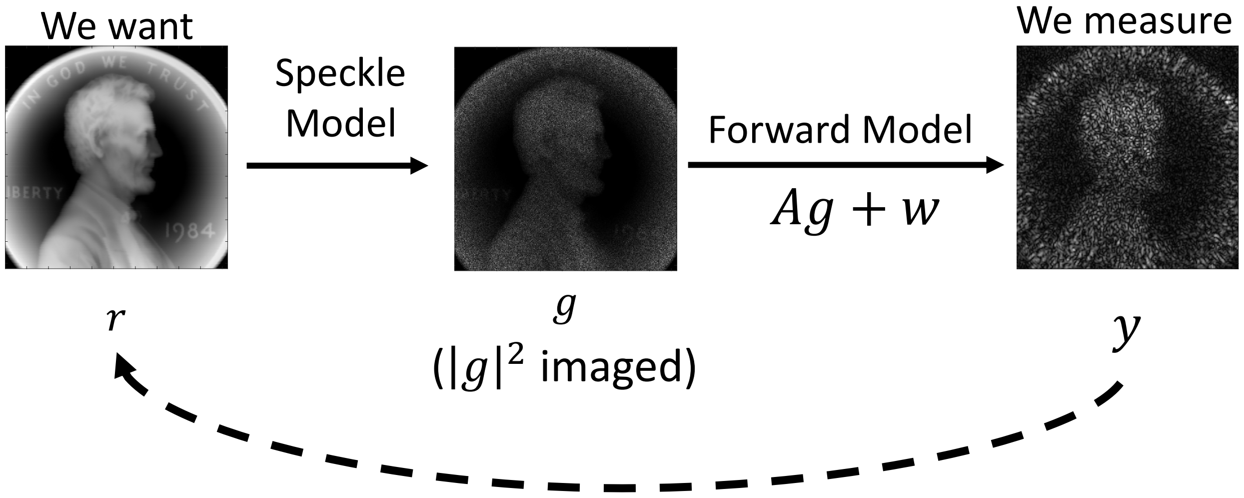
$A \in \mathbb{C}^{M \times N}$ - propagation matrix

$g \in \mathbb{C}^M$ - complex reflectance

$w \in \mathbb{C}^N$ - measurement noise (normal)



- Combine this with a statistical model of speckle
  - Ideally, we want $r = E[|g|^2]$ (infinite speckle realizations)
  - Great benefit == high spatial correlation



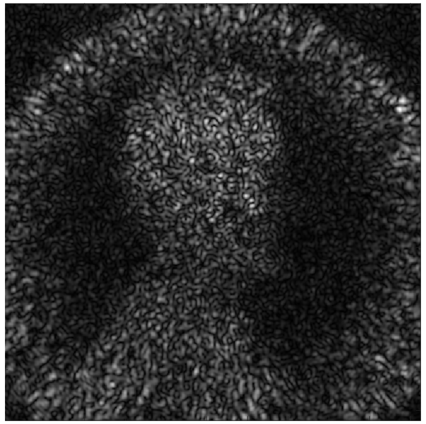
- Base inversion off MAP Estimate

$$\hat{r}_{MAP} = \arg\min_{r}\{-\log p(r|y)\}$$

Bayesian Inversion Background

Core Problem: How to reconstruct object of interest x from noisy measurements y (e.g. from a physical system)?

Obj. of interest: x



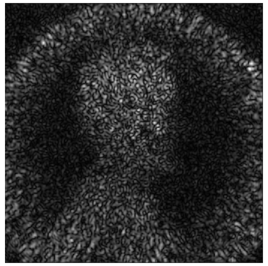
Measurements: y

Bayesian Inversion Background

Core Problem: How to reconstruct object of interest x from noisy measurements y (e.g. from a physical system)?

$x \longrightarrow$ Forward Model $\longrightarrow y$

Obj. of interest: x



Measurements: y

Bayesian perspective:

- A forward model describes distribution of the output y given the input x: $p(y|x)$

- A prior model describes distribution of the input x: $p(x)$

MAP Estimation:

$$\hat{x}_{MAP} = \arg\max_x \{p(x|y)\}$$
$$= \arg\min_x \{-\log p(y|x) - \log p(x)\}$$

Strike balance between fitting data & fitting prior knowledge

Regularized Inversion

$$\hat{x}_{MAP} = \arg\min_{x}\{\underbrace{-\log p(y|x)}\,\underbrace{-\log p(x)}\}$$

E.g. $y = Ax + w,\ \ w \sim N(0, \sigma_w^2 I)$

$$f(x) = \frac{1}{2\sigma_w^2}\|y - Ax\|^2$$

Least-squares data fit

E.g. $x \sim N(0, I)$

$$h(x) = \frac{1}{2}\|x\|^2$$

L_2-regularization

Regularized Inversion – Thin Manifold View

$$\hat{x} = \arg\min_{x}\{f(x) + h(x)\}$$

E.g. $y = Ax + w,\ \ w \sim N(0, \sigma_w^2 I)$

$$f(x) = \frac{1}{2\sigma_w^2}\|y - Ax\|^2$$

Least-squares data fit

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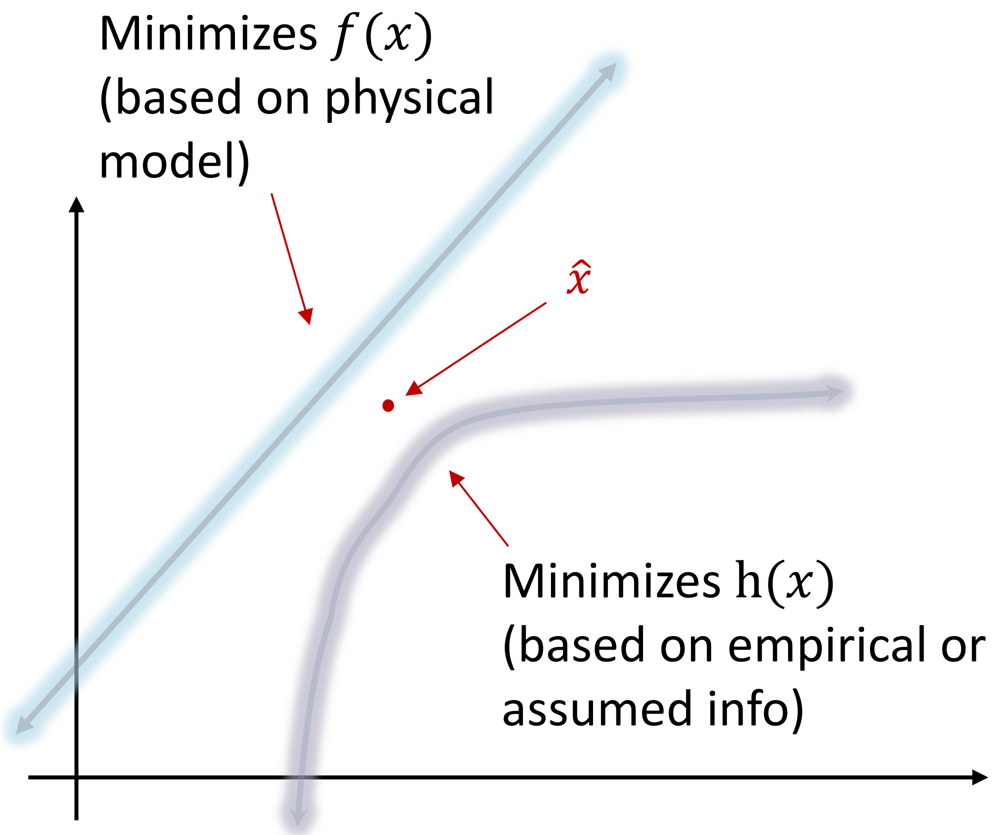
L_2-regularization



Variable Splitting

$$\hat{x} = \arg\min_{x=v}\{f(x) + h(v)\}$$

- Decouple optimization of $f(x)$ and $h(x)$

ADMM

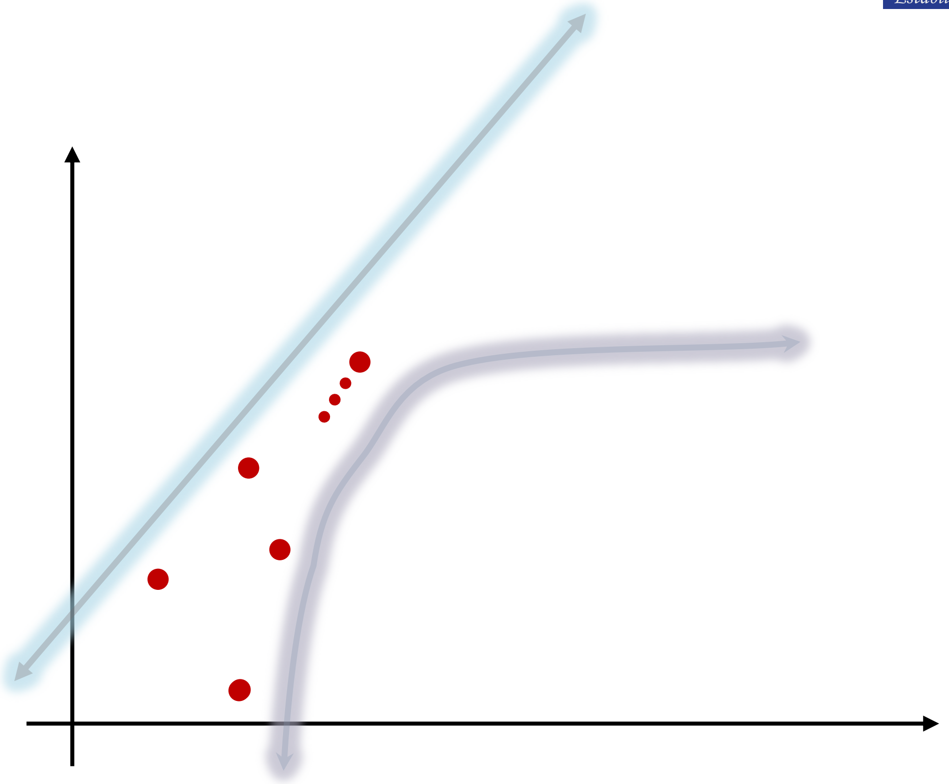
- Alternate minimization of $f(x)$ and $h(x)$ with augmented Lagrangian term for communication

- Uses **Proximal Maps**

$$\boxed{F(v) = \arg\min_{x}\{f(x) + \lambda\|x - v\|^2\}}$$

$$\boxed{H(x) = \arg\min_{v}\{h(v) + \lambda\|v - x\|^2\}}$$

Interpretation:
Take input, return something "better"



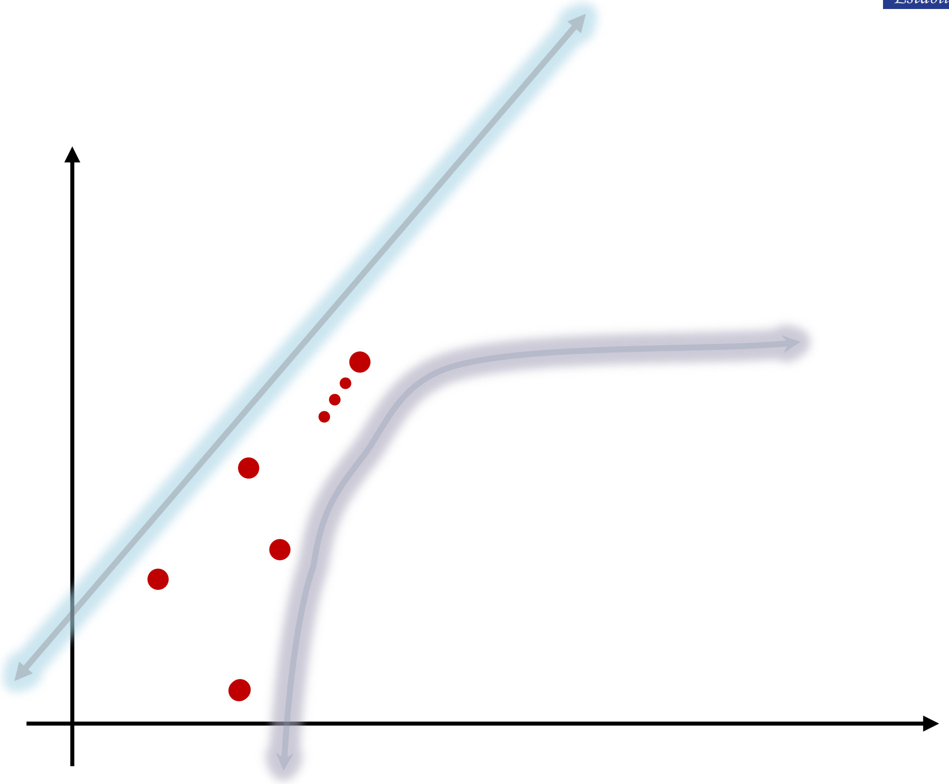
Variable Splitting

$$\hat{x} = \arg\min_{x=v}\{f(x) + h(v)\}$$

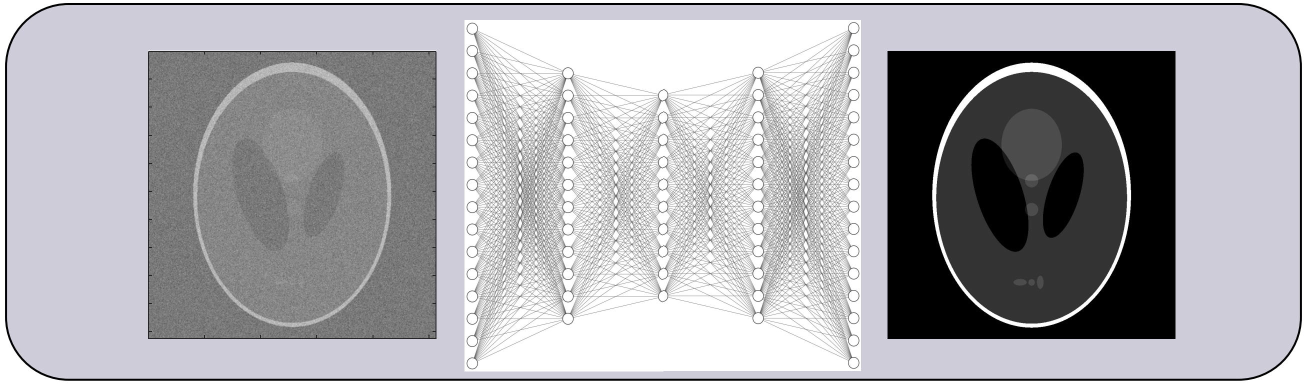
- Decouple optimization of $f(x)$ and $h(x)$

Plug and Play

- Replace proximal map(s) with algorithmic model(s), “agents” (e.g. Neural Networks, NLM, BM3D)



$$\boxed{F(v) = \arg\min_{x}\{f(x) + \lambda\|x - v\|^2\}}$$



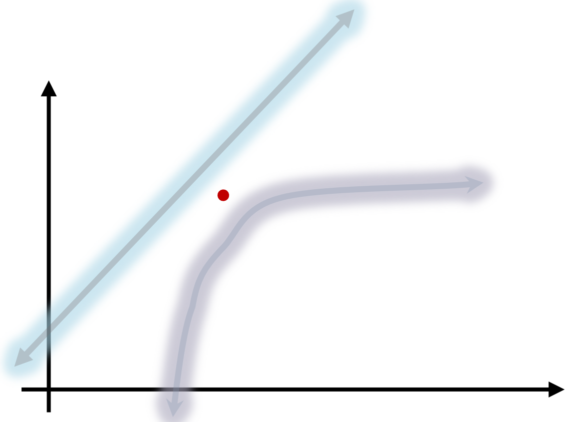
Interpretation:
Take input, return something “better”

Consensus Equilibrium

No longer solves optimization problem

$$\hat{x} = \arg\cancel{\min_{x}\{f(x)}+h(x)\}$$

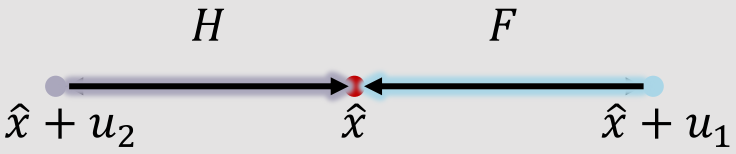
What problem are we solving?



Consensus Equilibrium conditions

Find $\hat{x}, u_1, u_2$ such that

$$\begin{cases} F(\hat{x}+u_1) = H(\hat{x}+u_2) = \hat{x} & \text{Consensus} \\ u_1 + u_2 = 0 & \text{Equilibrium} \end{cases}$$

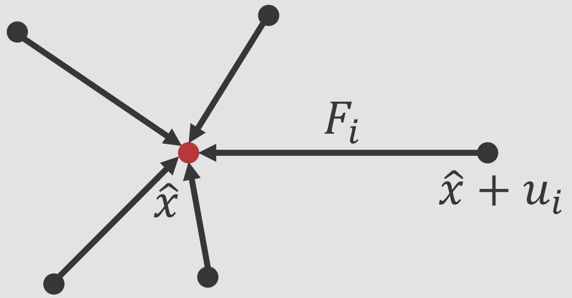


Multi-Agent Consensus Equilibrium

Consensus Equilibrium conditions

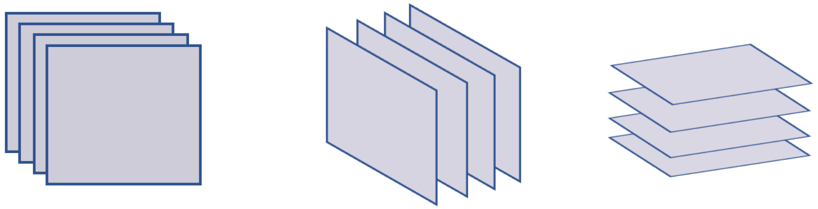
Find $\hat{x}, u_1, \ldots, u_n$ such that

$$\begin{cases} F_1(\hat{x} + u_1) = \cdots = F_n(\hat{x} + u_n) = \hat{x} & \text{Consensus} \\ u_1 + \cdots + u_n = 0 & \text{Equilibrium} \end{cases}$$

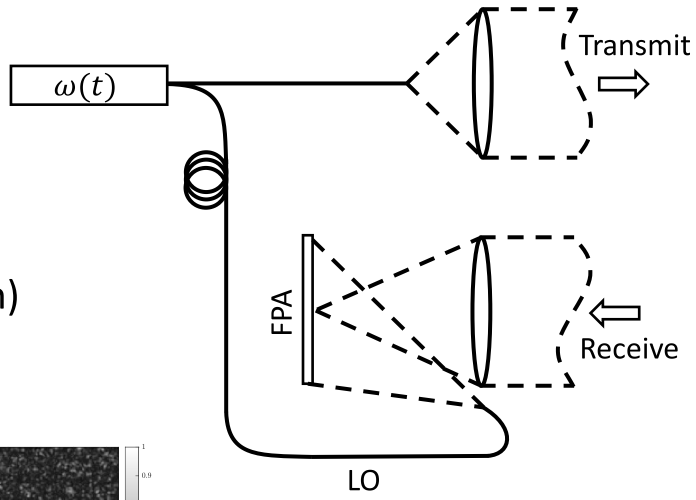


Benefit of Multiple Agents:

- Fit multiple data collections in parallel (better than speckle averaging)
- Combination of prior models (e.g. Multi Slice Fusion)

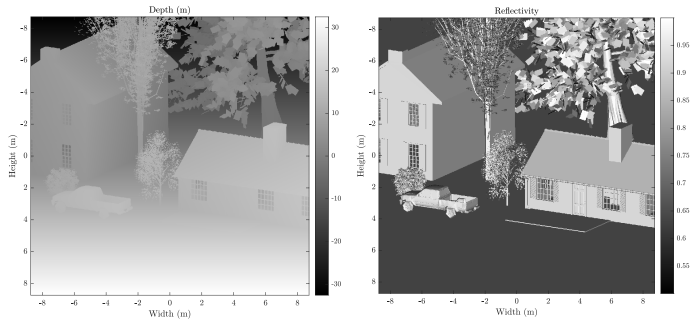


Simulated Experiment

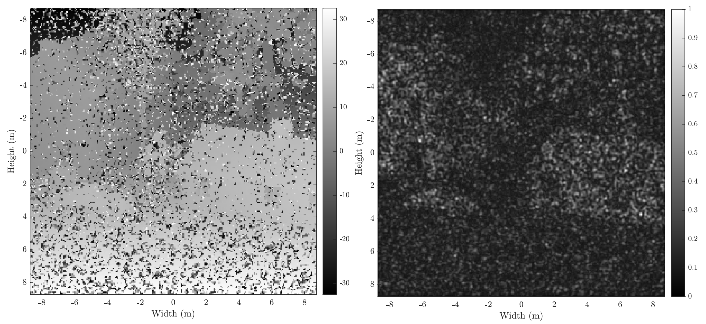


- **Simulated RCH detection under various conditions**
  - Number of Looks (speckle realizations)
  - SNR, characterized by mean photoelectrons per pixel (pre compression gain)
  - Note: **NO** turbulent phase errors modeled

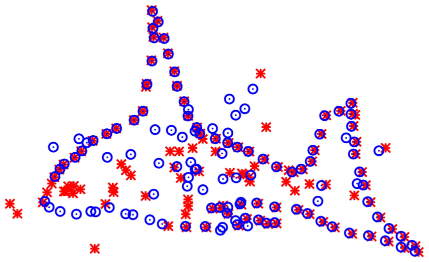
Ground Truth



Mean 2 pe's over 8 speckles



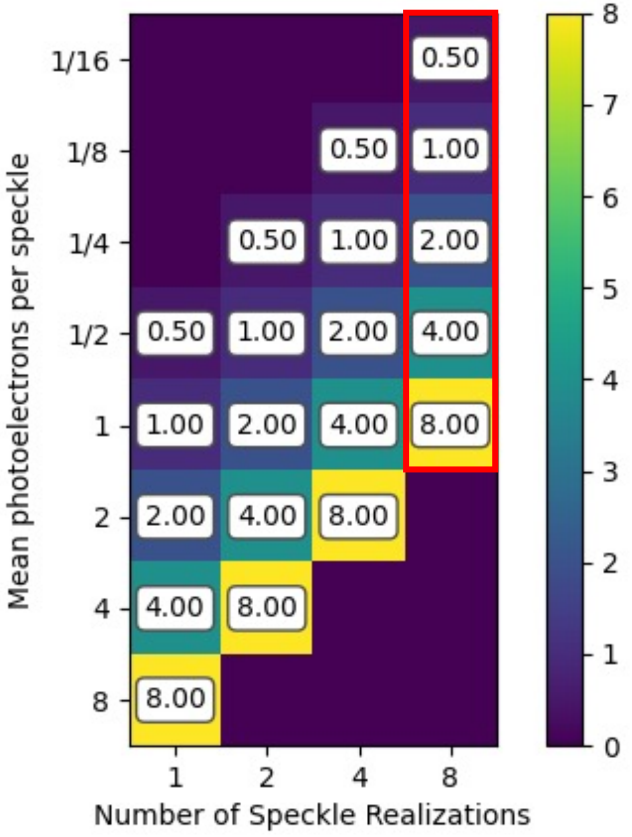
- **Quality Metric:** Point Cloud Root Mean Squared Error (PC-RMSE)
  - Convert 3D image to point cloud
  - Register with ground truth (match closest points)
  - Compute average distance
    - Drop distances > threshold (False Positive)



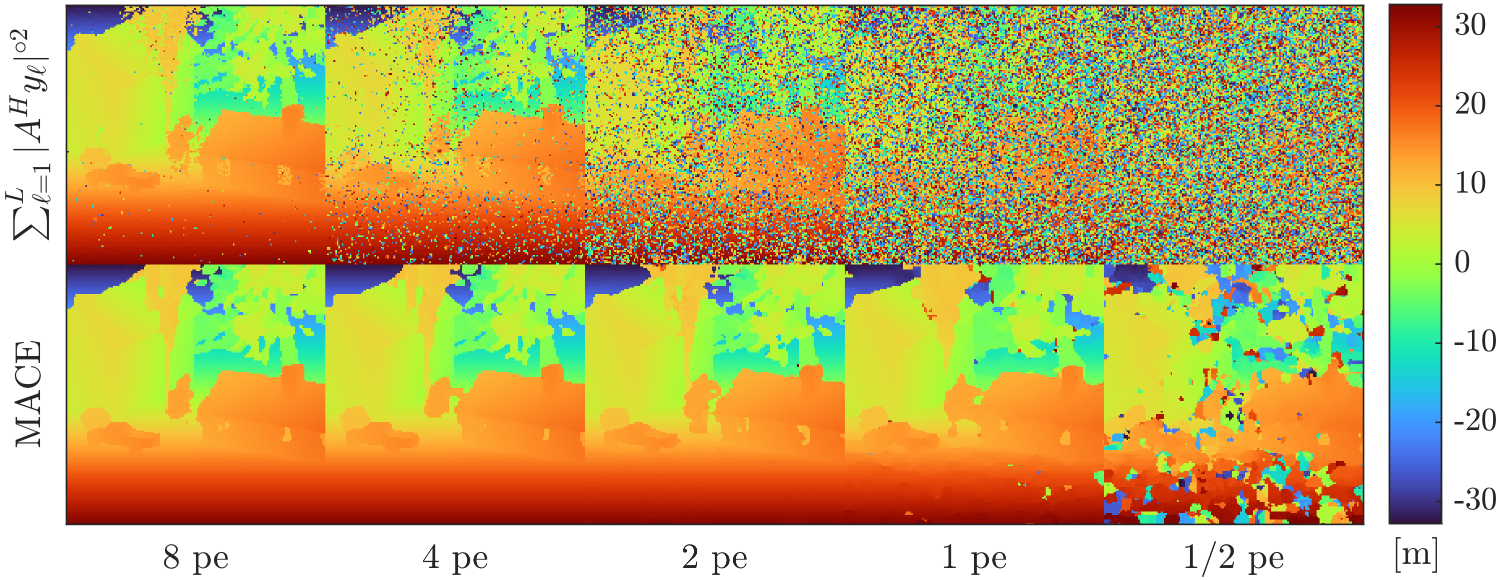
https://en.wikipedia.org/wiki/File:Cpd_fish_affine.gif

Results – Fixed # of Looks

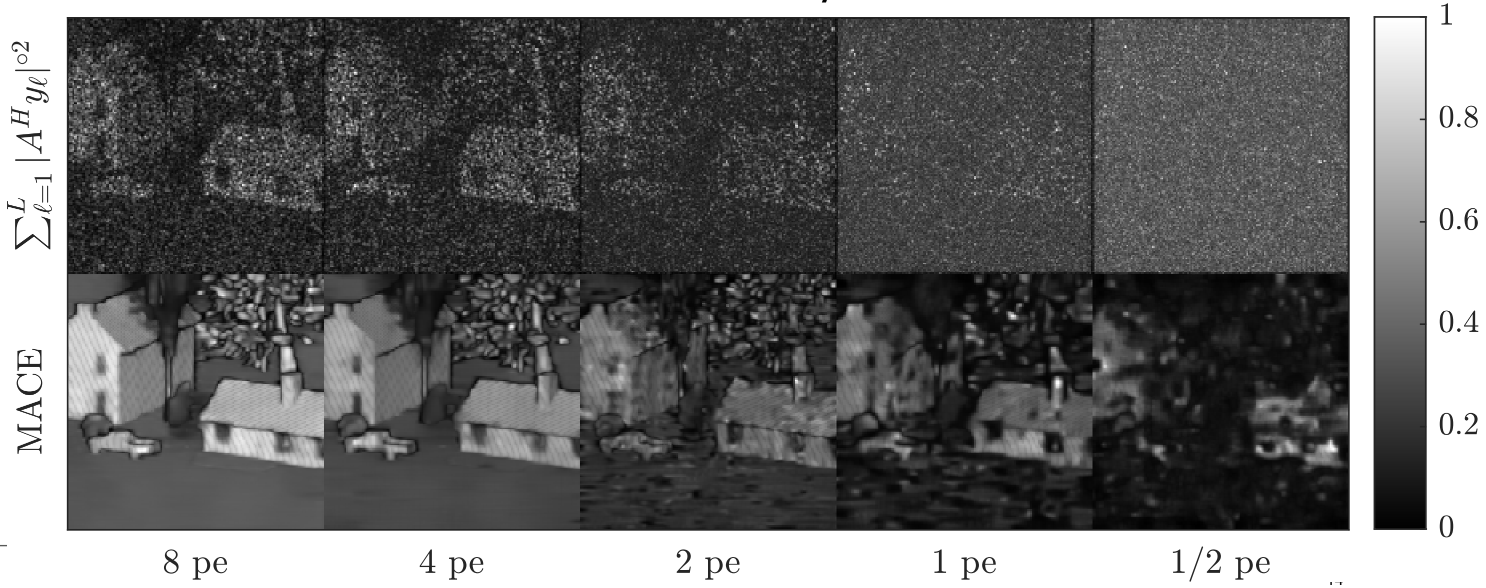
Total Photon Count



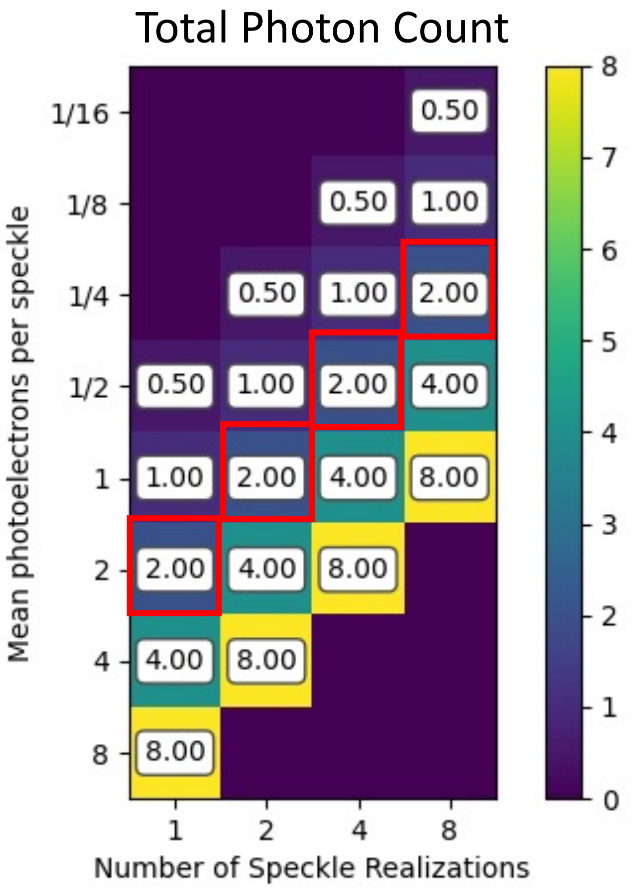
Depth Maps

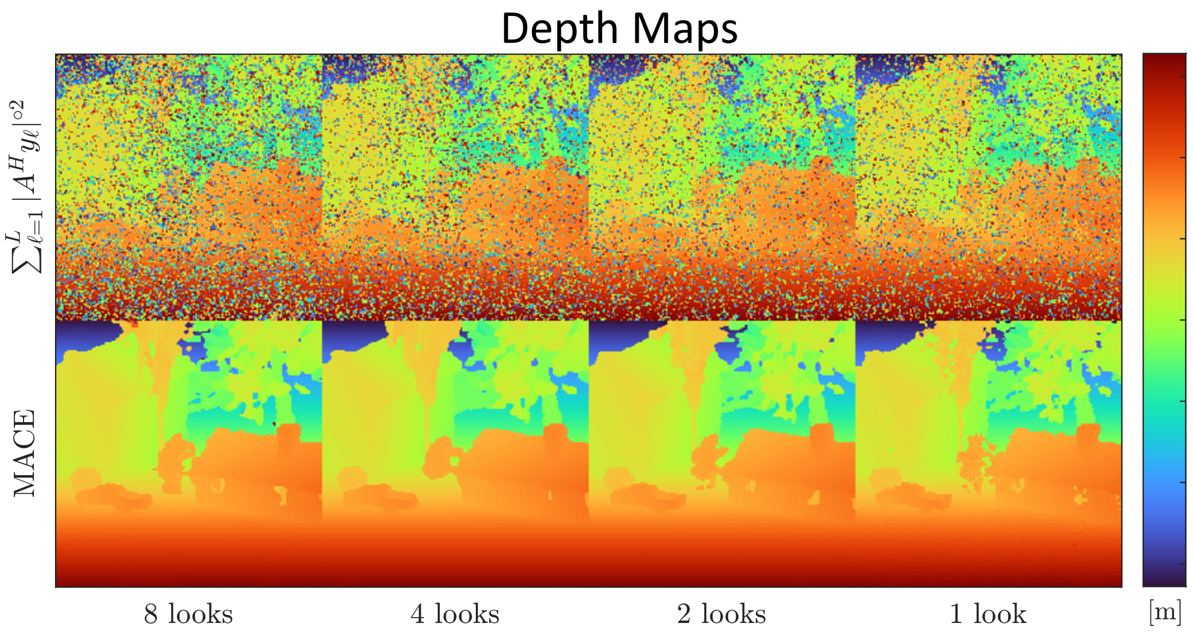


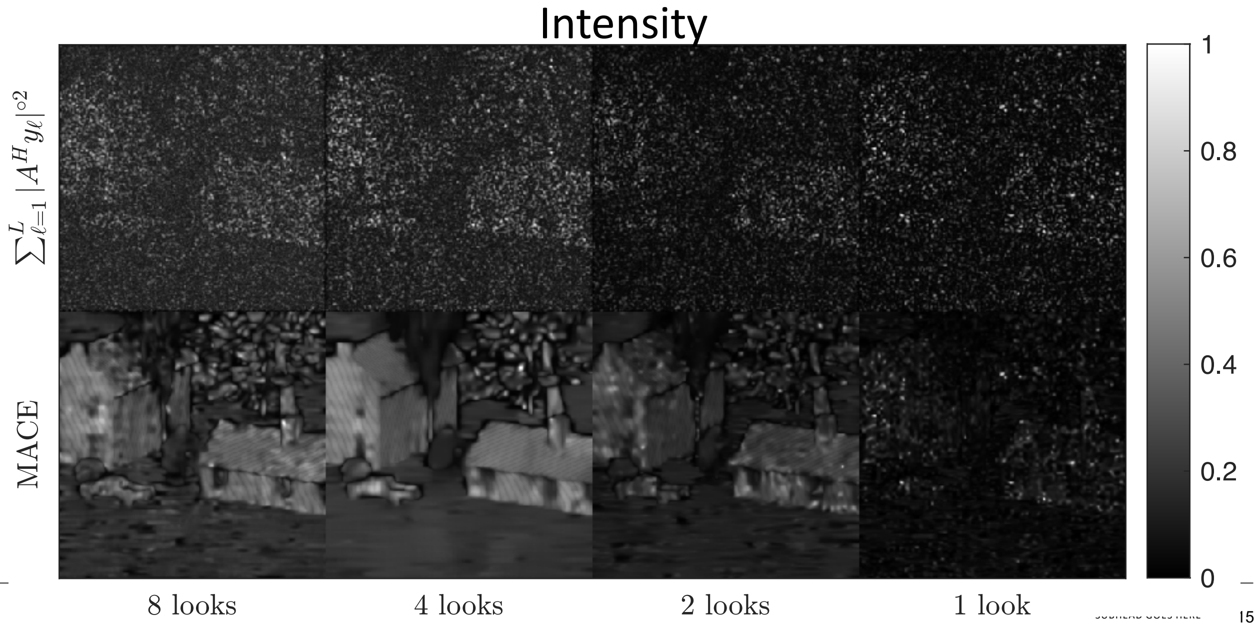
Intensity



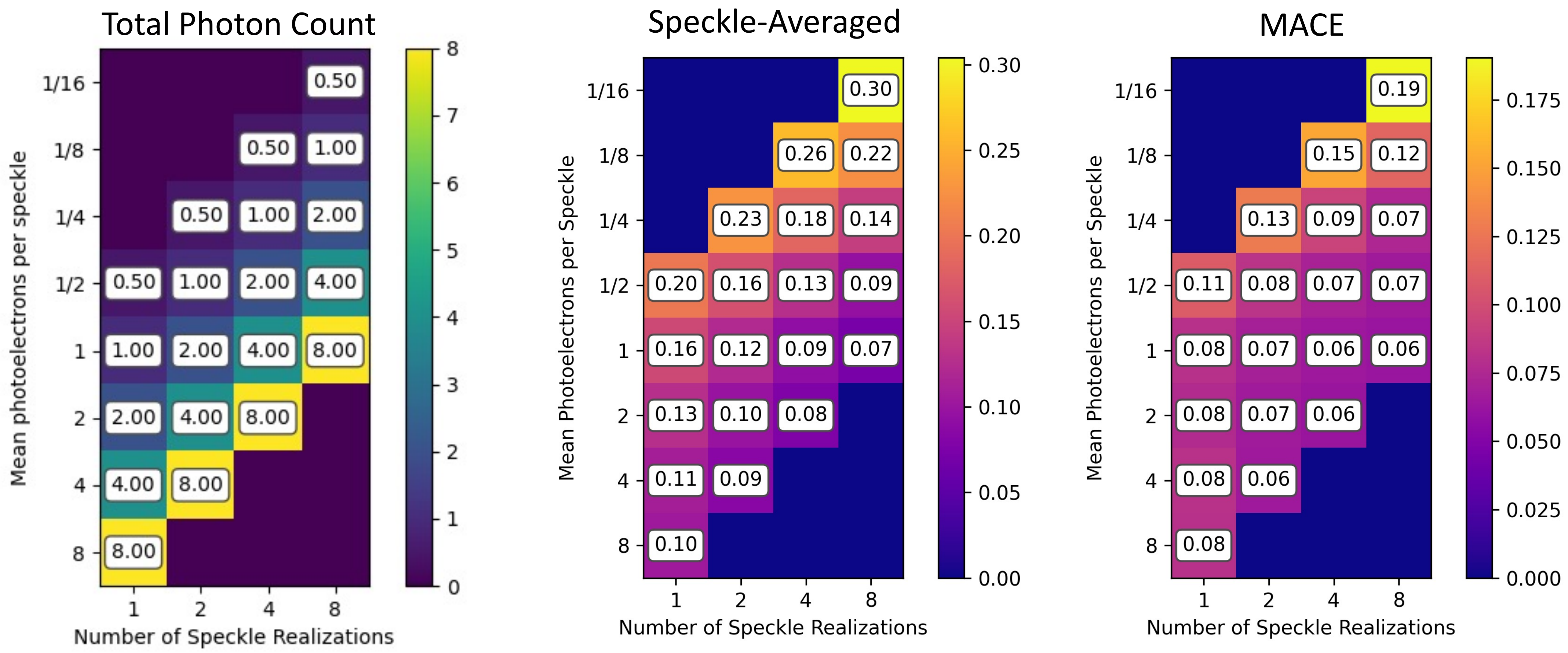
Results – Fixed total photons







Results – Point Cloud RMSE



QUESTIONS?

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