Research Statement

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My research focuses primarily on interactions between several areas of mathematics, including representation theory, algebraic geometry, commutative algebra, \(\mathcal{D}\)-modules and invariant theory. An overarching theme in my work is the geometry of algebraic varieties under the presence of symmetries, that is, varieties equipped with actions of algebraic groups. Such symmetries arise in many problems; for instance, whenever we are working with some coordinates “up to a change of basis”, it is natural to carry over the action of the general linear group and use its representation theory for various computations. Varieties that are endowed with actions of large groups include determinantal varieties, varieties of representations of quivers, nullcones, Vinberg representations arising from gradings on Lie algebras (e.g. symmetric- and skew-symmetric matrices, space of binary cubic forms, space of alternating senary 3-tensors), and other spaces with finitely many orbits (e.g. spherical varieties).

I enjoy investigating subtle geometric properties and calculating explicitly various invariants that are notoriously difficult to find. These include Bernstein-Sato polynomials, (minimal) generators and free resolutions of ideals, local cohomology groups, Lyubeznik numbers, geometric properties as normality, Cohen-Macaulay, or rational singularities.

First, I introduce some basics on quivers, as they appear in my work frequently. They arose originally from the study of finite dimensional algebras and their modules. A quiver \(Q\) is a finite oriented graph. A representation of \(Q\) is an assignment of vector spaces to each vertex and linear maps to each arrow. A quiver \((Q,R)\) (with relations) is a quiver \(Q\) together with a finite set of relations \(R\); a relation is a linear combination of paths in \(Q\). A representation \(V\) of \((Q,R)\) is a representation of \(Q\) such that the linear maps of \(V\) satisfy the relations imposed by \(R\).

In the following sections I present briefly the central topics of my research.

1 Equivariant \(\mathcal{D}\)-modules and applications

1.1 Introduction

For simplicity, assume \(X = \mathbb{C}^d\) is the complex affine space, equipped with the action of an algebraic group \(G\). Let \(\mathcal{D}_X\) denote the ring of differential operators, or the Weyl algebra. It is generated as \(\mathcal{D}_X = \mathbb{C}(x_1,\ldots,x_d,\partial_1,\ldots,\partial_d)\), with relations \([\partial_i,x_j] = \delta_{ij}\), where \(\delta_{ij}\) is the Kronecker delta. The action of \(G\) induces some vector fields on \(X\), hence a map \(g \to \mathcal{D}_X\), where \(g\) is the Lie algebra of \(G\). A \(\mathcal{D}_X\)-module \(M\) is (strongly)
equivariant if the action of \( \mathfrak{g} \) on \( M \) via the map \( \mathfrak{g} \to \mathcal{D}_X \) can be integrated to an algebraic \( G \)-action on \( M \).

Equivariant \( \mathcal{D}_X \)-modules arise naturally in many ways. For example, the coordinate ring \( \mathbb{C}[X] \) or its localization at a semi-invariant polynomial, local cohomology of the \( \mathbb{C}[X] \) in a (locally) closed \( G \)-stable subvariety give rise to equivariant \( \mathcal{D}_X \)-modules. On the other hand, equivariant \( \mathcal{D} \)-modules are quite rigid when the group \( G \) is large.

When \( G \) acts on \( X \) with finitely many orbits, any coherent equivariant \( \mathcal{D}_X \)-module is automatically regular holonomic (see [HTT08, Theorem 11.6.1])—this means that it has “smallest” possible size. Moreover, in this case there are only finitely many simple\( \mathcal{D} \)-modules. It seems a reasonable task to classify and describe explicitly all equivariant \( \mathcal{D} \)-modules in such cases, which in general is considered to be a difficult problem [MV86, Open Problem 3].

### 1.2 Categories of equivariant \( \mathcal{D} \)-modules

Let \( \text{mod}_G(\mathcal{D}_X) \) denote the category of equivariant coherent \( \mathcal{D}_X \)-modules. In [LW18a], we give a systematic study of \( \text{mod}_G(\mathcal{D}_X) \) (e.g. [LW18a Propositions 1.2, 1.4]). When \( G \) acts on \( X \) with finitely many orbits, the category \( \text{mod}_G(\mathcal{D}_X) \) is equivalent to the category of finite-dimensional representations of a quiver with relations. This follows by the Riemann-Hilbert correspondence from the analogous result proved for perverse sheaves in [MV86, Vil94]. We give a more elementary and constructive proof of this result for \( \mathcal{D} \)-modules [LW18a, Theorem 3.4]. This constructive approach, together with various techniques developed in [LW18a], allows us to tackle the problem of finding the explicit description of the quiver (with relations) in many cases.

One such case is when \( X \) is a representation of a linearly reductive group \( G \), and \( B \) acts on \( X \) with a dense orbit, where \( B \) is a Borel subgroup of \( G \). In [LW18a], we call such a space \( X \) a \( G \)-spherical vector space. In this setting, we show that several beautiful phenomena occur: any equivariant simple \( \mathcal{D} \)-module is multiplicity-free as a \( G \)-module (for some explicit decompositions, see [Rai16]), and its characteristic cycle is also multiplicity-free [LW18a, Theorem 3.16, Corollary 3.18]. Moreover, when the \( G \)-module \( X \) is irreducible, we find explicitly the quivers corresponding to \( \text{mod}_G(\mathcal{D}_X) \) [LW18a, Theorem 5.2]. In almost all cases, the quiver of \( \text{mod}_G(\mathcal{D}_X) \) is a disjoint union of quivers of type

\[
\widehat{A}_n : \quad (1) \rightarrowtail (2) \rightarrowtail \ldots \rightarrowtail (n-1) \rightarrowtail (n) ,
\]

where all the 2-cycles are relations. In fact, the quiver \( \widehat{A}_n \) has (up to isomorphism) only finitely many indecomposable representations, and they can be described explicitly [LW18a, Theorem 2.11]. Only for the representation corresponding to \( \text{Sp}_4 \otimes \text{GL}_4 \) the quiver is the “doubling” of a different Dynkin quiver (see [LW18a, Theorem 5.2. (b)]):

\[
\widehat{E}_6 : \quad \begin{array}{c}
\beta \\
\alpha
\end{array}
\]

\[
(1) \rightarrowtail (2) \rightarrowtail (3) \rightarrowtail (4) \rightarrowtail (5)
\]
where all the compositions with $\alpha$ or $\beta$ are relations, as well as all 2-cycles. Though not of finite representation type as $\tilde{\mathbb{A}}_n$, the quiver $\tilde{\mathbb{E}}_6$ is tame [LW18a Theorem 2.14].

We investigate similar problems beyond spherical spaces as well. On the space of binary cubic forms $\text{Sym}_3 \mathbb{C}^2$, in [LRW17] we described all the simple equivariant $\mathcal{D}$-modules and obtained the quiver of $\text{mod}_G(\mathcal{D}_X)$, whose largest connected component is the quiver

\[
\begin{array}{c}
(1) \\
\downarrow \\
(2) \\
\downarrow \\
(3) \\
\downarrow \\
(4) \\
\end{array}
\]

where all 2-cycles and all non-diagonal compositions of two arrows are relations. This quiver is again of tame representation type [LRW17 Theorem 4.4]. Similarly, we describe all the simple equivariant $\mathcal{D}$-modules on the space of alternating senary 3-tensors $\bigwedge^3 \mathbb{C}^6$ [LP18] and on the other representations of the so-called subexceptional series, where the quivers are a disjoint union of two type $\tilde{\mathbb{A}}_3$ quivers as in (1). This subexceptional series of representations is related to Freudenthal's magic square, see [LM04].

An indispensable ingredient in these results is the Bernstein-Sato polynomial (see [LW18a Proposition 4.9]). We present details about these polynomials in Section 3.

1.3 Applications to local cohomology

Exploiting our results on equivariant $\mathcal{D}$-modules, we determine some local cohomology modules with support in orbit closures, expressing them as direct sums of indecomposable $\mathcal{D}$-modules. Computing local cohomology modules is a difficult problem in commutative algebra and algebraic geometry, and in the case of finitely many orbits we demonstrate that the study of equivariant $\mathcal{D}$-modules provides a very powerful tool in obtaining results that are completely explicit.

In [LR18], we compute the (iterated) local cohomology modules with support in determinantal varieties. Some results were known in this direction (e.g. [RW14]), but these did not provide a complete picture on the structure of local cohomology modules. Let $X$ be the space of $m \times n$ matrices, with $m \geq n$. Let $O_p$ denote the variety of matrices of rank $p$, which is an orbit under the action $G = \text{GL}_m \times \text{GL}_n$. Then $X = \bigcup_{p=0}^{p} O_p$ has finitely many orbits under the action of $G$. There are exactly $n + 1$ simple equivariant coherent $\mathcal{D}_X$-modules $D_0, \ldots, D_n$, where the support of $D_p$ is $\overline{O}_p$. Note that $D_n = \mathbb{C}[X]$ is the coordinate ring.

For $a \geq b \geq 0$ we define the Gaussian binomial coefficient

\[
\binom{a}{b}_q = \frac{(1 - q^a) \cdot (1 - q^{a-1}) \cdots (1 - q^{a-b+1})}{(1 - q^b) \cdot (1 - q^{b-1}) \cdots (1 - q)}.
\]

Setting $q \mapsto q^2$, this gives the Poincaré polynomial of $\text{Grass}(b, a)$. We have:
Theorem 1.1 ([LR18] Theorem 1.1]). For every $0 \leq t < p \leq n \leq m$ we have the following in the Grothendieck group of $\text{mod}_G(\mathcal{D}_X)$:

$$\sum_{j=0}^{t} [H^j_{\mathcal{O}_t}(D_p)] \cdot q^j = \sum_{s=0}^{t} [D_s] \cdot q^{(p-t)^2+(p-s) \cdot (m-n)} \cdot \binom{n-s}{p-s} \cdot \binom{p-1-s}{t-s} q^2.$$ 

As it turns out, in the non-square case $m \neq n$, the category $\text{mod}_G(\mathcal{D}_X)$ is semi-simple [LW18a, Theorem 5.4 (b)]. Hence, the above formula is actually a direct sum decomposition, and it gives all iterated local cohomology modules $H^j_{\mathcal{O}_t}(D_p) = H^j_{\mathcal{O}_{t+1}}(\mathcal{O}_{t+1}) \cdot \cdots \cdot H^j_{\mathcal{O}_t}(D_p)$.

In the square case $m = n$, the situation is more complicated, as the category $\text{mod}_G(\mathcal{D}_X)$ is given by the quiver $\mathbb{A}_{n+1}$ as in [1]. Let $S_{\det}$ denote the localization of $S = \mathbb{C}[X]$ at the determinant. The Bernstein-Sato polynomial of $\det$ (see [3]) provides a filtration of $S_{\det}$, from which we obtain the following indecomposable equivariant $\mathcal{D}$-modules $Q_p$ (with the corresponding representation of $\mathbb{A}_{n+1}$)

$$Q_p := \frac{S_{\det}}{(\det^p-n+1)}, \quad \mathbb{C} \xrightarrow{1} \mathbb{C} \xrightarrow{1} \cdots \xrightarrow{1} \mathbb{C} \xrightarrow{0} \mathbb{C} \xrightarrow{0} \mathbb{C} \xrightarrow{0} \cdots \xrightarrow{0} 0 \quad (p \ 1's)$$

Let $\text{add}(Q)$ denote the subcategory of $\text{mod}_G(\mathcal{D}_X)$ formed of $\mathcal{D}$-modules that are direct sums of $Q_0, Q_1, \ldots, Q_{n-1}$. In [LR18, Section 6] we show that for $t < p \leq n$, we have $H^j_{\mathcal{O}_t}(D_p) \in \text{add}(Q)$, and we also have $H^j_{\mathcal{O}_t}(Q_p) \in \text{add}(Q)$ $(p \neq n)$. Moreover, we obtain explicit $\mathcal{D}$-module direct sum decompositions for these local cohomology modules [LR18, Section 6]. Hence, we determine the direct sum decompositions of the iterations $H^j_{\mathcal{O}_{t+1}}(H^j_{\mathcal{O}_{t+2}}(\cdots H^j_{\mathcal{O}_t}(D_p) \cdots))$ for the square case as well.

In particular, we obtain all the Lyubeznik numbers $\lambda_{i,j}(R^{(p)})$ for the determinantal rings $R^{(p)}$, thus answering a question of Melvin Hochster (for a survey on Lyubeznik numbers, see [NBWZ16]). For the square case $m = n$, we obtain:

Theorem 1.2 ([LR18, Theorem 1.5]). We have $\sum \lambda_{i,j}(R^{(n-1)}) \cdot q^j \cdot w^j = (q \cdot w)^{n^2-1}$ and for $0 \leq p \leq n-2$ we have

$$\sum_{i,j \geq 0} \lambda_{i,j}(R^{(p)}) \cdot q^j \cdot w^j = \sum_{s=0}^{p} q^{s^2+2s} \cdot \binom{n-1}{s} q^2 \cdot w^{p^2+2p+s(2n-2p-2)} \cdot \binom{n-2-s}{p-s} w^2.$$ 

Similarly, we give a complete description of all the (iterated) local cohomology modules supported at orbit closures and calculate the corresponding Lyubeznik numbers for binary cubic forms [LRW17] and alternating senary 3-tensors [LP18]. In [LW], we describe local cohomology on the other remaining representations from the subexceptional series, where many (but not all) of the results are uniform within the series.

2 Geometric study of the representation theory of quivers

2.1 Introduction

Let $Q$ be a quiver. For a fixed dimension vector $\alpha$, the set of representations with dimension vector $\alpha$ form an affine space $\text{Rep}(Q, \alpha)$ that is a direct product of spaces of
matrices. We investigate this space (and subvarieties) under the action of the product of general linear groups corresponding to the change of basis at each vertex. Under this action, there is a one-to-one correspondence between the isomorphism classes of representations $V$ and orbits $O_V$. The notion can be naturally extended to quivers with relations.

For an example, when $Q$ is of type $A_2$ with dimension vector $(m, n)$, orbits correspond to ranks, and the orbit closures are precisely the determinantal varieties of $m \times n$ matrices.

### 2.2 Orbit closures of quivers

The geometry of orbit closures $O_V$ has been studied intensively in various articles (for example [BZ01, BZ02, KR15, LM98, Zwa02]). In particular, it was shown that for Dynkin quivers of type $A$ and $D$ orbit closures have rational singularities, by reducing to the analogous facts on Schubert varieties. However, the type $E$ case is still open. Set-theoretic equations for orbit closures are known for all Dynkin quivers by [Bon96] and come from certain rank conditions. Nevertheless, finding the (minimal) defining equations and free resolutions of the orbit closures of Dynkin quivers is difficult in general.

We study these problems employing the geometric technique [Wey03] that is based on Kempf’s and Weyman’s generalization of Lascon’s free resolution for determinantal ideals. In the case of Dynkin quivers, Reineke [Rei03] provides desingularizations of orbit closures $O_V \subset \text{Rep}(Q, \alpha)$ as total spaces of some vector bundles over a product of flag varieties which we will denote by Flag. To such a desingularization we associate a locally free sheaf $\xi$, and apply in principle [Wey03, Basic theorem] to get a complex $F_{\bullet}$:

$$F_i = \bigoplus_{j \geq 0} H^j(\text{Flag}, \bigwedge^{i+j} \xi) \otimes A(-i - j). \quad (2)$$

Here $A$ denotes the coordinate ring of the representation space $\text{Rep}(Q, \alpha)$. When the complex $F_{\bullet}$ has no terms in negative degrees, the complex gives a minimal free resolution of the (normalization of the) coordinate ring of $O_V$ over $A$.

The case when calculations in the complex $F_{\bullet}$ are possible using Bott’s theorem is when the locally free sheaf $\xi$ is semi-simple. This happens for 1-step representations (see [Sut15], or a more general definition in [LW18b]), when Flag is just a product of Grassmannians. Generalizing the results in [Sut15], we give in [LW18b, Theorem 3.5] a beautiful connection between the geometry of 1-step orbit closures and the representation type of the quiver, on which the following result is based on.

#### Theorem 2.1 ([LW18b, Theorem 3.6]).

Let $Q$ be a Dynkin quiver and $V$ a 1-step representation. Then $O_V$ has rational singularities (and is thus also normal and Cohen-Macaulay) and $F_{\bullet}$ gives a minimal resolution of its coordinate ring.

We have a similar result for extended Dynkin quivers [LW18b, Theorem 3.7], when normality fails in general, but the normalization still has rational singularities. This suggests a geometric trichotomy parallel to the representation-theoretic one—quivers of finite, tame and wild type.
In [LW18b, Section 4] we describe explicitly the \textit{minimal} generators for the defining ideals of 1-step representations when the quiver is Dynkin of type $A$. Further, we show ([LW18b, Theorem 4.7]) that any representation can be written as a scheme-theoretic intersection of 1-step orbit closures. This provides an algorithm for finding an efficient generating set for the defining ideal of any orbit closure of a type $A$ quiver.

In [Lör], we consider similar questions for the equioriented $A_3$ quiver. In this classical case, orbit closures correspond to varieties of pairs of matrices $(A, B)$ such that $\text{rank}(A) \leq a$, $\text{rank}(B) \leq b$ and $\text{rank}(A \cdot B) \leq c$, for some $a, b, c$ with $c \leq \min\{a, b\}$. In this situation not all representations are 1-step. Hence, we cannot choose in general $\xi$ to be semi-simple in (2), and Bott’s theorem is not applicable directly. Nevertheless, we reduce in [Lör] the study of minimal free resolutions of orbit closures to the calculation of the cohomology of vector bundles on the 2-step flag $\text{Flag}(r_1, r_2, n)$ of the form

$$S_\lambda R_2 \otimes S_\mu (W/R_1)^*,$$

where $S_\lambda$ (resp. $S_\mu$) denotes the Schur functor corresponding to the partition $\lambda$ (resp. $\mu$), $R_1$ (resp. $R_2$) is the tautological subbundle of dimension $r_1$ (resp $r_2$), and $\text{dim } W = n$. In [Lör] we provide methods to compute these cohomology spaces using Schur complexes (see [Wey03]), which is of independent interest.

2.3 Representation varieties of algebras with nodes

The operation of node splitting for finite-dimensional algebras was introduced in [MV80]. Let $Q$ be a quiver and consider its path algebra $\mathbb{C}Q$. A \textit{node} of an algebra $A = \mathbb{C}Q/I$ is a vertex $x$ of $Q$ such that all the paths of length 2 passing strictly through $x$ belong to the ideal $I$. A node $x$ of $A$ can be \textit{split} by the following local operation around $x$ obtaining naturally an algebra $A^x = \mathbb{C}Q^x/I^x$:

The representation theory of $A$ and $A^x$ are very closely related. In [KL18], we present various results on the relations between the geometry of varieties of representations in $A$ and $A^x$. In fact, we show that there is a natural map between varieties of representations from $A^x$ to $A$ that preserves the property of normality and rational singularities [KL18, Theorem 1.2]. The strength of our method lies in working in the “relative setting”, i.e. splitting locally a node without assuming any restrictions on the rest of the algebra. This is a very useful result for studying geometry of varieties of representations, as nodes are quite common in the representation theory of quivers (for example, each quiver in Section 1.2 has several nodes).

Results are particularly strong in the case of radical square zero algebras, which are algebras for which every vertex is a node. Splitting its nodes repeatedly we can describe
all of its irreducible components in a purely combinatorial way [KL18, Theorem 4.3] and show that they have mild singularities.

**Theorem 2.2.** [KL18, Corollary 1.4] Let $A$ be a finite-dimensional $\mathbb{C}$-algebra with $\text{rad}^2 A = 0$. Then for any dimension vector $\alpha$, any irreducible component $C \subseteq \text{Rep}(A, \alpha)$ has rational singularities (and is thus also normal, and Cohen-Macaulay).

By results in [CK18], normality of the irreducibly components of $\text{Rep}(A, \alpha)$ has strong consequences on the decomposition of its moduli spaces of its semistable representations in relation with Geometric Invariant Theory [KL18, Section 5].

Our results are applicable to a wide class of algebras that we illustrate by numerous examples [KL18, Examples 4.1, 4.6, 4.9, 4.11, 4.12, 5.4].

3 Bernstein-Sato polynomials

3.1 Introduction

Bernstein-Sato polynomials were introduced by J. Bernstein and M. Sato independently in the early seventies and have applications to singularity theory, monodromy theory etc. (for a survey, see [Bud]). Let $f \in \mathbb{C}[x_1, \ldots, x_d]$ be a non-zero polynomial. The definition is based on the following existence result:

**Definition 1.** There is a differential operator $P(s) \in D_X[s] := D_X \otimes \mathbb{C}[s]$ and a non-zero polynomial $b(s) \in \mathbb{C}[s]$ such that

$$P(s) \cdot f^{s+1}(x) = b(s) \cdot f^s(x).$$

Among such polynomials we call the (monic) polynomial $b(s)$ of smallest degree the Bernstein-Sato polynomial (or $b$-function) of $f$.

In general, the computation of $b$-functions of arbitrary polynomials is a difficult task. In the equivariant setting, several techniques have been developed. One case is that of prehomogeneous vector spaces (see [ST77]), that is, spaces that have a dense open orbit under the action of an algebraic group (indeed, this case was the original motivation for M. Sato). A semi-invariant is a polynomial invariant under the action of the group up to a character. An example in this setting is Cayley’s classical identity that gives the $b$-function of the determinant:

$$\det(\partial) \det(X)^{s+1} = (s + 1)(s + 2) \cdots (s + n) \det(X)^s. \quad (3)$$

A lot of effort has been made to calculate $b$-functions of semi-invariants of prehomogeneous spaces (for example, see [Kim82, SKKO80]). In case of quivers, we call a dimension vector prehomogeneous if the corresponding representation space is prehomogeneous. For Dynkin quivers, each dimension vector is prehomogeneous. Semi-invariants of quivers have been studied extensively [DW00, Sch91, SdB01], and they are (sums of) determinants of matrices formed of suitable block matrices of variables.
3.2 b-functions of semi-invariants of quivers

In [Lör18, Lör17] we give two different techniques for the computation of the b-functions (of several variables) of various semi-invariants. For the case of quivers, both techniques generalize results obtained for type A quivers in [Sug11].

Our first technique is a slice method ([Lör18, Theorem 2.15]) that realizes a decomposition of the Bernstein-Sato polynomial of a semi-invariant into the product of two Bernstein-Sato polynomials - on associated to an ideal, the other to a smaller semi-invariant. This is based on our generalization ([Lör18, Theorem 2.5]) of a representation-theoretic multiplicity one property that was previously studied in [SS06]. In fact, this can be applied in other contexts and gives an elementary approach for the determination of the b-function of the determinant, symmetric determinant, Pfaffian and other classical invariants (see [Lör18, Section 2.3]). The technique is efficient for computing the b-functions of many semi-invariants, including for tree quivers of “small weights” [Lör18, Theorem 3.13, Theorem 3.14].

The other technique is based on relating b-functions under the so-called reflection functors (also known as Coxeter or castling transformations) fundamental to representation theory (see [ASS06]). For quivers, this is an operation performed at a sink (resp. source) vertex by changing the orientation of all the arrows ending (resp. starting) at the vertex. We generalize a formula in [Kim82] and obtain a relation between the b-functions of semi-invariants related under reflection functors [Lör17, Theorem 2.1, 2.2, 4.1]. Namely, if we put

\[
[s]_{\alpha_1}^{d_1}, \ldots, [s]_{\alpha_k}^{d_k} := \prod_{i=1}^{a} \prod_{j=0}^{d_i-1} (d_1 s_1 + \cdots + d_k s_k + i + j),
\]

then the following holds.

**Theorem 3.1** ([Lör17, Theorem 4.1]). Let Q be quiver with a prehomogeneous dimension vector β and \( f_i \in \text{SI}(Q, \beta)_{(\alpha_i, \cdot)} \) be semi-invariants, where \( i = 1, \ldots, k \). Assume the coordinates of \( c(\beta) \) are non-negative. Then the b-function satisfies the formula

\[
b_f(s) = b_c(f_j)(s) \prod_{x \in Q_0} \frac{[s]_{\beta_x}^{c(\alpha_1)_x \cdots, c(\alpha_k)_x}}{[s]_{\beta_x}^{c(\alpha_1)_x \cdots, c(\alpha_k)_x}}.
\]

This allows, in particular, the computation of b-functions (of several variables) for all Dynkin quivers [Lör17, Corollary 4.3] as well as extended Dynkin quivers with prehomogeneous dimension vectors [Lör17, Proposition 4.5]. For examples, see [Lör17, Example 4.6, 4.7, 4.8, 4.9].

3.3 Singularities of zero sets of semi-invariants for quivers

Bernstein-Sato polynomials provide fine numerical invariants of singularities (for example, cf. [BMS06, Sai93]). Using our Theorem 3.1 and representation theory of quivers,
we have show that codimension 1 orbit closures in for Dynkin quivers have rational singularities \[\text{[Lör15, Theorem 3.5]}\] and give a similar result for extended Dynkin quivers \[\text{[Lör15, Theorem 3.7]}\].

In \[\text{[BMS06]}\] the notion of the Bernstein-Sato polynomial has been generalized to arbitrary varieties. In \[\text{[Lör15, Proposition 3.2]}\] we give a basic relation between the \(b\)-function of several variables of some semi-invariants and the Bernstein-Sato polynomial of their zero set. Using this, we give some results when nullcones have rational singularities \[\text{[Lör15, Proposition 3.10, Example 3.12]}\]. In order to prove these claims using Bernstein-Sato polynomials, one has to work with reduced complete intersections \[\text{[BMS06, Theorem 4]}\]. It was shown in \[\text{[RZ03]}\] that nullcones indeed become (irreducible) complete intersections when the prehomogeneous dimension vector is not “too small”. We prove that they also become reduced:

**Theorem 3.2** (\[\text{[Lör15, Theorem 2.7]}\]). Let \(Q\) be a quiver and \(\alpha\) a prehomogeneous dimension vector. Then there is a positive integer \(N\) such that the nullcone in \(\text{Rep}(Q, n \cdot \alpha)\) is reduced for any \(n \geq N\).

We give a bound for \(N\) in \[\text{[Lör15, Theorem 2.10]}\] for Dynkin quivers that is sharp, as we found a type \(E_8\) quiver where the nullcone is not reduced \[\text{[Lör15, Example 2.12]}\].

### 3.4 Bernstein-Sato polynomial for maximal minors

Conjectures have been made on the Bernstein-Sato polynomials of minors of a generic \(m \times n\) matrix of variables (see \[\text{[Bud]}\]). These have been disproved in general, but we confirmed them in the case of maximal minors \[\text{[LRWW17]}\]. Let \(m \geq n\). We show

**Theorem 3.3** (\[\text{[LRWW17, Theorem 4.1]}\]). The Bernstein-Sato polynomial of the ideal of maximal minors equals \((s + m - n + 1) \cdots (s + m)\).

There are two main steps in the proof of this result. The first part is finding the equation giving the predicted Bernstein-Sato polynomial. We have done this in the more general setting of so-called multiplicity-free tuples \[\text{[LRWW17, Proposition 3.4]}\]. From this we computed the predicted polynomial (see \[\text{[LRWW17, Theorem 3.5]}\]). In \[\text{[LRWW17, Section 4]}\] we prove that the predicted polynomial indeed is minimal, therefore it is the Bernstein-Sato polynomial. This is the more difficult part of the proof.

The method works also for the case of sub-maximal Pfaffians \((n \text{ odd}):\)

**Theorem 3.4** (\[\text{[LRWW17, Theorem 3.9]}\]). The Bernstein-Sato polynomial of the ideal of sub-maximal Pfaffians equals \((s + 3)(s + 5) \cdots (s + n)\).

We also give connections between roots of Bernstein-Sato polynomials and the non-vanishing of certain local cohomology groups \[\text{[LRWW17, Corollary 3.2]}\].

Finally, we conclude that the Strong Monodromy Conjecture holds for maximal minors and sub-maximal Pfaffians \[\text{[LRWW17, Section 5]}\].
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