Math 460 "Cheat Sheet"

Basic Facts

(BF1) SSS: Three sides determine a triangle up to congruence.
(BF2) SAS: Two sides and an included angle determine a triangle up to congruence.
(BF3)ASA: Two angles and an included side determine a triangle up to congruence.
BF4: Ratios of corresponding sides for two similar triangles are the same. (The definition of similar is that the angles are the same.)
BF5: If two lines are crossed by a transversal, then: if the lines are parallel the corresponding angles are the same; if two corresponding angles are the same, lines are parallel.
BF6: Lengths, angles and areas add up.
BF7: Through two points there is exactly one line.
BF8: On a ray there is exactly one point at a given distance from the endpoint.
BF9: A line segment extends to a line. (Line segments finite, lines are infinite in both directions.)
BF10: Line segments have midpoints.
BF11: Angles have bisectors.
BF12: It is possible to find line perpendicular to a given line through a given point.
BF13: It is possible to fine a line parallel to a given line through a point not on the line.
BF14: Two lines parallel to a third line are parallel to each other.
BF15: The area of a rectangle is base times height.

Some Major Theorems

Theorem 1: When two lines cross, adjacent angles add up to 180 degrees. Vertical angles are equal.
Theorem 2: Suppose that two lines l and m are crossed by a transversal. a) l and m are parallel if and only if alternate interior angles are equal. b l and m are parallel if and only if each pair of interior angles add up to 180 degrees.
Theorem 3: Sum of angles of a triangle are 180 degrees.
Theorem 5: Opposite sides of triangle are equal if and only if opposites angles are equal. (Such a triangle is isoceles.)
Theorem 7: The area of triangle is one half base times height.
Theorem 8: If $\Delta ABC \sim \Delta DEF$ and the ratio $AB/DF = r$, then the area of the first triangle is r^2 times the area of the second.
Theorem 9: (Pythagorean theorem). The square of the hypotenuse of a right angle triangle is the sum of the squares of the sides.
Theorem 10: If two right triangles have the hypotenuse and leg matching, then they are congruent.
Theorem 11,12: Given a parallelogram (which means opposite sides are parallel), the opposites sides (thm 11) and angles (thm 12) are equal.
Theorem 13: If a pair of sides are equal and parallel, then it's a parallelogram.
Theorem 14: A quadralateral is a parallelogram if and only if diagonals bisect each other.
Theorem 17, 18: (Thm 17) In a triangle ABC, let D be a midpoint of AC and suppose E is a point of BC with DE parallel to AB. Then E is a midpoint of BC and DE = AB/2. (Thm 18) Conversely, if E is a midpoint of BC, then DE is parallel to AB and DE = AB/2.
Theorem 19, 20: In triangles ABC abd DEF, (Thm 19) if $\angle C = \angle F$ and $AC/DF = BC/EF$ or (Thm 20) if $AB/DE = AC/DF = BC/EF$, then they are similar.
Theorem 22: The area of a triangle ABC is $\frac{1}{2}AB \cdot AC \cdot \sin \angle A$. (If $\angle A$ is part of right triangle, then $\sin \angle A$ is given by $opposite/hypotenuse$.)
Theorem 23: In a triangle ABC, $\frac{\sin \angle A}{BC} = \frac{\sin \angle B}{AC} = \frac{\sin \angle C}{AB}$
Theorem 24: The perpendicular bisectors of a triangle are concurrent. (The point where they meet is the circumcenter.)
Theorem 25. Given a triangle with circumcenter O, suppose that a circle with center O that goes through one of the vertices of the triangle. Then it also goes through the other two vertices.
Theorem 26: The angle bisectors of a triangle are concurrent. (The point where they meet is called the incenter.)
Theorem 27: The altitudes of a triangle are concurrent. (The point where they meet is called the orthocenter.)

Theorem 28: The point where two medians meet is 2/3 of the way from the vertices.
Theorem 29: The medians of a triangle are concurrent. (The point where they meet is called the centroid.)
Theorem 30: The circumcenter, centroid and orthocenter of a triangle are collinear.
Theorem 31,34: (Menelaus' theorem). Let ABC be a triangle, A' a line on BC etc, so that A', B', C' colinear then

$$\frac{A'B}{A'C}\frac{B'C}{B'A}\frac{C'A}{C'B} = 1$$

(for theorem 34, the signed ratio

$$\frac{\vec{A'B}}{\vec{A'C}}\frac{\vec{B'C}}{\vec{B'A}}\frac{\vec{C'A}}{\vec{C'B}} = 1$$

)
Theorem 35: (Converse to Menelaus). Let ABC be a triangle, A' a line on BC etc, so that

$$\frac{\vec{A'B}}{\vec{A'C}}\frac{\vec{B'C}}{\vec{B'A}}\frac{\vec{C'A}}{\vec{C'B}} = 1$$

then A', B', C' are colinear.
Theorem 36,37: (Ceva's theorem and converse) Let ABC be a triangle, A' a line on BC etc. If AA', BB',CC' are concurrent, then

$$\frac{\vec{A'B}}{\vec{A'C}}\frac{\vec{B'C}}{\vec{B'A}}\frac{\vec{C'A}}{\vec{C'B}} = -1$$

Conversely, if this equation holds then AA', BB',CC' are concurrent.
Theorem 38: A, B, C are points on circle with center O. If B is outside $\angle AOC$, then $\angle ABC = \frac{1}{2}\angle AOC$. If B is inside, then $\angle ABC = 180° - \frac{1}{2}\angle AOC$.
Theorem 40. Let A, B, C be points on a circle, and let ADC be the arc cut off by $\angle ABC$. Then $\angle ABC = \frac{1}{2} arc\, ADC$.
Theorem 43. Let A be a point on the circle with center O, and let m be a line through A. Then m is tangent to the circle if and only if m is perpendicular to OA.
Theorem 44. It is possible to construct a circle with given center tangent to a given line.
Theorem 45. Given a triangle with incenter I, the circle with center I tangent to one of the sides is tangent to the other two.
Theorem 46. Let A and B be points on a circle, let C be a point on the tangent line at B, and let ADB be the arc cut off by $\angle ABC$. Then $\angle ABC = \frac{1}{2} arc\, ADB$.
Theorem 47. If the opposite pairs of angles in a quadrilateral add to 180° then there is a circle going through all four vertices.
Theorem 100: Midpoints of spherical line segments exist.
Theorem 101: Given a spherical line ℓ and a point P not on the line, there exists a line m passing through P which meets ℓ at right angles ($\pi/2$ radians).
Theorem 102: Given two points, there exists a spherical line containing them. If the points are antipodes (opposite) then then there are infinitely many lines, otherwise there is one.
Theorem 103: Any two spherical lines meet.
Theorem 104: If ΔABC is a spherical triangle, $\angle A + \angle B + \angle C = \pi + area(\Delta ABC)$
Theorem 106: Suppose that ℓ is a spherical line and P is a point not on ℓ. If P a pole of ℓ, then for any point Q on ℓ, the spherical distance $PQ = \pi/2$. Suppose that for two points Q_1, Q_2 on ℓ we have $PQ_1 = \pi/2$ and $PQ_2 = \pi/2$ then P is a pole of ℓ. Suppose that P is a pole of ℓ and Q_1, Q_2 are on ℓ, then $Q_1Q_2 = \angle Q_1PQ_2$.

n Theorem 107: If $\Delta A'B'C'$ is the polar triangle to ΔABC, then ΔABC is the polar triangle to $\Delta A'B'C'$. ($\Delta A'B'C'$ is the triangle with A a pole of $B'C'$ on the same side as A' etc.)
Theorem 108: If $\Delta A'B'C'$ is the polar triangle to ΔABC, then $\angle A + B'C' = \pi$.
Theorem 109-110: Spherical AAA, ASA holds.
Theorem 112: Let ΔABC be a spherical triangle with $\angle C = \pi/2$. Let a, b, c be the lengths of sides opposite to A, B, C. Then $\cos c = \cos a \cos b$; $\cos \angle A = \frac{\cos a \sin b}{\sin c}$; $\sin \angle A = \frac{\sin a}{\sin c}$
Theorem 113: Let ΔABC be a spherical triangle with sides a, b, c opposite A, B, C. Then $\cos c = \cos a \cos b + \sin a \sin b \cos C$
Theorem 114-115: SSS, SAS hold for spherical triangles.