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REGULARITY OF THE STOKES OPERATOR IN THIN DOMAINS

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The problem arises in the study of the Navier-Stokes equations with Navier friction boundary conditions in a 3D thin domain Ω_ε which has thickness of order $O(\varepsilon)$ as $\varepsilon \to 0$, and non-flat top and bottom boundaries. Our aim is to obtain the estimate $\|u\|_{H^2(\Omega_\varepsilon)} \le C_\varepsilon \|Au\|_{L^2(\Omega_\varepsilon)}$, with explicit dependence of C_ε on ε, where u belongs to the domain D_A of the Stokes operator A which is related to the mentioned boundary conditions. Because of the boundary's non-trivial geometry and the involved boundary conditions, the constant C_ε is not known to have a uniform bound when $\varepsilon \to 0$. Our result is the following estimate

$$\|u\|_{H^2(\Omega_\varepsilon)} \le c\|Au\|_{L^2(\Omega_\varepsilon)} + c_\varepsilon \|u\|_{L^2(\Omega_\varepsilon)}, \quad u \in D_A,$$

where c is independent of ε, the positive number c_ε depends explicitly on ε and the friction coefficients, and may blow up as $\varepsilon \to 0$. (This estimate is used in the theory of the global strong solutions to the Navier-Stokes equations considered above.)