THE 3RD SYMPOSIUM ON ANALYSIS AND PDES

PURDUE UNIVERSITY, MAY 27-30, 2007

SHARP FRIEDRICHS-KNAPP-STEIN INEQUALITIES AND APPLICATIONS

JUAN J. MANFREDI, UNIVERSITY OF PITTSBURGH, PITTSBURGH, PA

The classical Friedrichs identity states that for $u\in C_0^\infty(\mathbb{R}^n)$ we have

$$\int_{\mathbb{R}^n} |D^2 u|^2 \, dx = \int_{\mathbb{R}^n} |\Delta u|^2 \, dx$$

From this inequality we immediately get $W^{2,2}$ -estimates for solutions of $\Delta u = f$ and also for solutions of measurable perturbations of the form $\sum_{ij} a_{ij}(x)u_{ij}(x) = f(x)$, when the matrix $A = (a_{ij})$ is closed to the identity in sense made precise by Cordes. In this talk we explore extensions of the Friedrichs identity in the form of sharp inequalities

$$\int_X |\mathfrak{X}^2 u|^2 \, dx \le C_1 \int_X |\Delta_{\mathfrak{X}} u|^2 \, dx + C_2 \int_X |\mathfrak{X}|^2 \, dx$$

where X is a Riemannian manifold, a nilpotent Lie group, and a CR manifold.

This is joint work with Andràs Domokos (PAMS 133, 2005) and Sagun Chanillo (2007 preprint.)