THE 5TH SYMPOSIUM ON ANALYSIS & PDES

PURDUE UNIVERSITY, MAY 20–23, 2012

TITLES & ABSTRACTS

PRINCIPAL LECTURERS

Luis Caffarelli, *University of Texas at Austin*

MINICOURSE. Fractional obstacle problem: optimal regularity

Abstract. We will discuss the obstacle problem for fractional Laplacian: basic properties of the solution, initial regularity, monotonicity formula for convex solutions and optimal regularity, the Almgren monotonicity formula and stable points, regularity of the free boundary in a neighborhood of such points.

Alice Chang, *Princeton University*

MINICOURSE. Nonlinear PDEs in the study of conformal invariants

– LECTURE I. *Conformal invariants: perspectives from geometric PDE.*

We will survey properties of a class of integral conformal invariants conformal geometry and their connection to geometric quantities on conformally compact Einstein manifolds in ADS/CFT setting. Special emphasis will be on the role played by non-linear elliptic PDE.

– LECTURES II & III. *Higher order isoperimetric inequalities: an approach via method of optimal transport.*

One of the method to derive sharp isoperimetric inequality for domains in the Euclidean is to apply the method of optimal transport; in this talk, I will report some recent joint work with Yi Wang to extend the method to prove some higher order isoperimetric inequalities with weights involving symmetric functions of the second fundamental form.

INVITED SPEAKERS

Giovanna Citti, *Universita di Bologna*

Heat kernels and mean curvature flow in Lie groups with subelliptic metric

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Matthew Gursky, *University of Notre Dame*

Regularized determinants and conformally invariant operators

Abstract. In this talk I will give an overview of some geometric and analytic issues related to the regularized determinant of an elliptic operator. I will begin with a quick overview of the work of Osgood-Phillips-Sarnak on the determinant of the laplacian for surfaces, then move to four dimensions, where the starting point is a formula of Branson-Orsted for conformal variations of the determinant. I will talk about a question posed by Connes concerning the determinant of the Paneitz operator and the half-torsion, and describe some variational properties, including multiplicity results.

YanYan Li, *Rutgers University*

Some analytic aspects of conformally invariant fully nonlinear equations

Abstract. We will discuss some work on conformally invariant elliptic and degenerate elliptic equations arising from conformal geometry. These include results on Liouville type theorems, Harnack inequalities, and Bocher type theorems.

Jean Michel Roquejoffre, *Universite de Toulouse*

Travelling graphs for the forced mean curvature motion in arbitrary space dimensions

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Lenya Ryzhik, *Stanford University*

On the Bramson logarithmic correction for KPP fronts

Abstract. I will recall a well known result of Bramson on the location of solutions of the KPP equation with compactly supported initial data. Bramson has originally proved this result using probabilistic techniques (large deviation methods). I will describe a simple PDE proof of a (weaker version) of Bramson's theorem, and explain how it can be generalized to periodic media. This is a joint work with F. Hamel, J. Nolen and J.-M. Roquejoffre.

Ovidiu Savin, *Columbia University*

Regularity of non-local minimal cones in dimension 2

Abstract. We show that the only nonlocal s-minimal cones in dimension two are the trivial ones for all parameters s in $(0, 1)$. We also discuss our methods in other more general settings. This is a joint work with E. Valdinoci.

Natasa Sesum, *Rutgers University*

Ancient solutions to geometric flows

Abstract. We discuss ancient solutions in different geometric flows. We focus on the classification of ancient solutions in the curve shortening flow and the Ricci flow on surfaces. We also construct a new ancient solution to the Yamabe flow on $\mathbb{R}^n$, which is equivalent to the fast diffusion equation on $\mathbb{R}^n$. The last is joint work with Daskalopoulos and del Pino.

Dejan Slepcev, *Carnegie Mellon University*

A nonlocal functional from data analysis

Abstract. I will discuss a nonlocal functional relevant to analysis of data sets. Specifically to nonlinear principal component analysis, data parameterization and dimensionality reduction. The basic features of the functional and connections to the average-distance problem introduced by Buttazzo, Oudet and Stepanov will be presented. The (lack of) regularity of minimizers will also be discussed.

Alexis Vasseur, *University of Texas at Austin*

Regularity theory for non-linear integral differential operators

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Paul Yang, *Princeton University*

Conformally covariant equations in 3-D CR geometry

Abstract. I will report on progress in a family of problems about conformally invariant equations in CR geometry, that has bearing on imbedding, CR-Yamabe problem and a formulation of a sphere theorem in CR geometry.

Yu Yuan, *University of Washington*

Hessian estimates for special Lagrangian equations with critical and supercritical phases

Abstract. We talk about a priori Hessian estimates for special Lagrangian equation with critical and supercritical phases in general higher dimensions. The "gradient" graphs of solutions are minimal Lagrangian submanifolds. Our unified approach leads to sharper estimates even for the previously known three dimensional or convex solution cases. Recent counterexamples for subcritical phase equations will also be mentioned.

This is joint work with Dake Wang.

Contributed Speakers

Mark Allen, *Purdue University*

The two-phase thin obstacle problem

Abstract. The thin obstacle problem can be reformulated in terms of minimizing a new functional. A two-phase problem then naturally arises which has applications in heat control at the boundary of a domain. This is analogous to the two phase classical obstacle problem which has applications in heat control through the interior of a domain. We study both the regularity of the solutions and the free boundaries.

Marian Bocea, *Loyola University Chicago*

Models for growth of heterogeneous sandpiles via Mosco convergence

Abstract. I will discuss the asymptotic behavior of variable exponent power-law functionals in the framework of Mosco convergence, and indicate some consequences to the analysis of a class of quasilinear parabolic problems which in the fast/slow diffusion limit models the growth of sandpiles whose critical slopes depend explicitly on the position in the sample. This is joint work with M. Mihailescu, M. Perez-Llanos, and J.D. Rossi.

Marie Frentz, *Umeå University*

The obstacle problem for parabolic non-divergence form operators of Hörmander type

Abstract. In this paper we establish the existence and uniqueness of strong solutions to the obstacle problem for a class of parabolic sub-elliptic operators in non-divergence form structured on a set of smooth vector fields in $\mathbb{R}^n$, $X = \{X_1, \dots, X_q\}$, $q \leq n$, satisfying Hörmander's finite rank condition. We furthermore prove that any strong solution belongs to a suitable class of Hölder continuous functions. As part of our argument, and this is of independent interest, we prove an a priori interior estimate, valid in the context of Sobolev spaces defined in terms of the system of vector fields.

This is joint work with E. Götmark and K. Nyström

Michael Goldman, *Ecole Polytechnique/Carnegie Mellon*

Differentiability and strict convexity of the stable norm

Abstract. I will show how the cell formula for the stable norm (or minimal action) recently proved by Chambolle and Thouroude permits to investigate its strict convexity and differentiability properties. For totally irrational directions it is proven that the stable norm is always differentiable whereas in

the other directions it is differentiable if and only if the plane-like minimizers in this direction which satisfy the strong Birkhoff property give rise to a foliation of the space. This answers a conjecture raised by Caffarelli and De La Llave. It is a joint work with A.Chambolle and M.Novaga.

Tao Huang, *University of Kentucky*

On uniqueness of heat flow of harmonic maps and hydrodynamic flow of nematic liquid crystals

Abstract. Abstract: We establish the uniqueness of the heat flow of harmonic maps that have sufficiently small renormalized energies. As corollaries, we obtain (i) the uniqueness of heat flow of harmonic maps whose gradients belong to $L_t^p L_x^q$ for $q > n$ and (p, q) satisfying Serrin's condition, and (ii) the uniqueness for hydrodynamic flow (u, d) of nematic liquid crystals, with $u, \nabla d$ satisfying Serrin's condition.

Phuc Nguyen, *Louisiana State University*

Stationary Navier-Stokes equations with critically singular external forces: existence and stability results

Abstract. We show the unique existence of solutions to stationary Navier-Stokes equations with small singular external forces belonging to a critical space. To the best of our knowledge, this is the largest critical space that is available up to now for this kind of existence. This result can be viewed as the stationary counterpart of the existence result obtained by H. Koch and D. Tataru for the free non-stationary Navier-Stokes equations with initial data in BMO^{-1}. The stability of the stationary solutions in such spaces is also obtained by a series of sharp estimates for resolvents of a singularly perturbed operator and the corresponding semigroup. This talk is based on joint work with Tuoc Van Phan.

Dake Wang, *University of Washington*

Singular solutions to special Lagrangian equations with subcritical phases and minimal surface systems

Abstract. We construct singular solutions to special Lagrangian equations with subcritical phases and minimal surface systems. A priori estimate breaking families of smooth solutions are also produced correspondingly.

Jiuyi Zhu, *Wayne State University*

Liouville-type theorems and decay estimates for solutions to higher order elliptic equations

Abstract. Liouville-type theorems are powerful tools in partial differential equations. Boundedness assumption of solutions are often imposed in deriving such Liouville-type theorems. We establish some Liouville-type theorems without the boundedness assumption of nonnegative solutions to certain classes of elliptic equations and systems. Using a rescaling technique and doubling lemma developed recently, we improve several Liouville-type theorems in higher order elliptic equations, some semi-linear equations and elliptic systems. More specifically, we remove the boundedness assumption of the solutions which is required in the proofs of the corresponding Liouville-type theorems in the recent literature. Moreover, we also investigate the singularity and decay estimates of higher order elliptic equations.