6th Symposium on Analysis and PDEs

Purdue University, June 1–4, 2015

Soledad Benguria, University of Wisconsin-Madison *June 3, 4:00–4:25pm*

The Brezis-Nirenberg Problem on $\mathbb{S}^n$, in Spaces of Fractional Dimension.

We consider the nonlinear eigenvalue problem,

$$-\Delta_{\mathbb{S}^n} u = \lambda u + |u|^{4/(n-2)} u,$$

with $u \in H_0^1(\Omega)$, where Ω is a geodesic ball in $\mathbb{S}^n$ contained in a hemisphere. In dimension 3, Bandle and Benguria proved that this problem has a unique positive solution if and only if

$$\frac{\pi^2 - 4\theta_1^2}{4\theta_1^2} < \lambda < \frac{\pi^2 - \theta_1^2}{\theta_1^2}$$

where θ_1 is the geodesic radius of the ball. For positive radial solutions of this problem one is led to an ODE that still makes sense when n is a real number rather than a natural number. We consider precisely that problem with $2 < n < 4$. Our main result is that in this case one has a positive solution if and only if λ is such that

$$\frac{1}{4}[(2\ell_2 + 1)^2 - (n-1)^2] < \lambda < 1[(2\ell_1 + 1)^2 - (n-1)^2]$$

where ℓ_1 (respectively ℓ_2) is the first positive value of ℓ for which the associated Legendre function $P_\ell^{(2-n)/2}(\cos\theta_1)$ (respectively $P_\ell^{(n-2)/2}(\cos\theta_1)$) vanishes. Joint work with Rafael D. Benguria (arXiv:1503.06347)