

1. Find the general solution to the equation

$$t \frac{dy}{dt} + 3y = 5t^2.$$

Find the solution that satisfies $y(1) = 4$.

2. Solve $y' = (x+1)y^2$, $y(0) = 2/3$. What is the interval of existence of the solution?
3. Find the general solution to the exact equation

$$(3x^2y^2 + 1) + (2x^3y + 2y) \frac{dy}{dx} = 0.$$

Express your solution y as an explicit function of x , if possible.

4. Solve the homogeneous equation

$$\frac{dy}{dx} = \frac{y}{x} + \frac{x}{y}.$$

Write your answer in the form $y =$ (a function of x).

5. Solve $2y' + y = e^{-t}$, $y(0) = y_0$. Describe the behavior of the solution as $t \rightarrow \infty$.
6. Let $y(t)$ satisfy

$$y' = y(y-1)^2(y+1)$$

Determine the asymptotic stability of the equilibrium solutions.

7. Consider the initial value problem $y' = xy + 3$, $y(1) = 3$. Find y_2 for the Euler approximation if $h = 2$.
8. Which of the following equations are linear? (Circle all that apply.)

- A. $xy' + (\sin x)y = y^2$
B. $xy' + (\sin x)y = x^2$
C. $\frac{d^2x}{dt^2} - (1-x^2)\frac{dx}{dt} + x = 0$
D. $x^2y'' + xy' + 2y = e^x$
E. $yy' + 6y = e^x$

9. What is the largest open interval for which a unique solution of the initial value problem

$$\frac{(t+1)(t-3)}{(t-2)}y' + \frac{(t-3)}{t(t-2)}y = \frac{(t+1)(t+3)}{t}, \quad y(1) = 0$$

is guaranteed?

- A. $0 < t < 1$ B. $0 < t < 2$ C. $0 < t < 3$ D. $1 < t < 3$ E. $-1 < t < 1$.

10. A body with a mass of 1 kg is thrown downward from a height of 5000 meters with an initial velocity 20 m/sec in a medium which offers resistance in proportion to twice the velocity. Find the equation of the height $x(t)$ of the body after t seconds. (Take $g = 9.8 \text{ m/sec}^2$.)

- A. $x(t) = 7.55e^{-2t} - 4.9t + 4992.5$
B. $x(t) = -4.9 - 15.1e^{-2t}$
C. $x(t) = 4.9t - 2502.5e^{-2t} + 2502.5$
D. $x(t) = -4.9t^2 - 20t + 5000$