1. Find the general solution to

$$y'' + 2y' + 5y = 20\cos x$$

by using the method of undetermined coefficients.

2. Find the solution to $y'' + 4y = 8x^2$ that satisfies $y(0) = 0$ and $y'(0) = 4$.

3. The homogeneous differential equation $t^2y'' - 4ty' + 6y = 0$ $(t > 0)$ has two solutions given by $y_1(t) = t^2$ and $y_2(t) = t^3$. Using the method of variation of parameters, find the general solution of the nonhomogeneous equation $t^2y'' - 4ty' + 6y = t^3$.

4. Find the general solution of

$$y''' + 4y'' + 5y' = 0.$$

5. Find the solution of the initial value problem

$$y^{(4)} + 2y'' + y = 3t + 4; \quad y(0) = y'(0) = 0, \quad y''(0) = y'''(0) = 1$$

by using the method of undetermined coefficients.

6. An object weighting 8 pounds attached to a spring will stretch it 6 inches beyond its natural length. There is a damping force with a damping constant $c = 6$ lbs-sec/ft and there is no external force. If at $t = 0$ the object is pulled 2 feet below equilibrium and then released, write the initial value problem describing the vertical displacement $x(t)$.

7. Find a particular solution, Y, of $y'' - 4y' + 3y = 2t + e^t$.

A. $Y = \frac{2}{3}t - \frac{8}{9} - \frac{2}{3}te^t$
B. $Y = \frac{2}{3}t + \frac{8}{9} - \frac{1}{2}te^t$
C. $Y = -2t - \frac{1}{2}te^t$
D. $Y = \frac{2}{3}e^t - \frac{1}{2}t$
E. $Y = \frac{2}{3}t + \frac{8}{9} + \frac{1}{3}te^t$

8. If $y'' + 5y' + 6y = 24e^t$, $y(0) = 0$, $y'(0) = 0$, then $y(1) =$?

A. $-8e^{-2} + 6e^{-3} + 2e$
B. $-e^{-2} - e^{-3} + 2e$
C. $12e - 24e^2 + 12e^3$
D. $3e^{-3} - 4e^{-2} + e$
E. $4e^{-2} + 3e^{-3} - 2e$

9. If $y_1(t) = t$ is a solution of $t^2y'' - 2ty + 2y = 0$, $t > 0$, use reduction of order to find a second solution $y_2(t)$

A. $y_2(t) = 1 + t$
B. $y_2(t) = 1 + t^2$
C. $y_2(t) = t + t^2$
D. $y_2(t) = t + t^3$
E. $y_2(t) = t^2 + t^3$

10. A certain spring-mass system leads to the initial value problem

$$2u'' + 6u = 8\cos\omega t, \quad u(0) = 0, \quad u'(0) = 2.$$

For what positive value of ω will resonance occur?

A. $\omega = 1/\sqrt{3}$ **B.** $\omega = \sqrt{6}$ **C.** $\omega = \sqrt{3}$ **D.** $\omega = 3$ **E.** $\omega = 8$

11. Using the method of undetermined coefficients, determine the *form* of the particular solution $Y(t)$ to the differential equation

$$y''' - y'' - y' + y = 2t + 3e^t.$$

A. $Y(t) = 2t + at^2e^t + bte^{-t}$

B. $Y(t) = at + b + ct^2e^t$

C. $Y(t) = at + be^t + cte^t + dt^2 + e^t$

D. $Y(t) = a + bt^2e^t$