1. Given $f(t) = \begin{cases} 1 & \text{if } 0 \le t < 1 \\ t & \text{if } 1 \le t \end{cases}$, determine $F(s) = \mathcal{L}\{f\}$.

A. $\dfrac{1}{s^2}$

B. $\dfrac{1}{s} + \dfrac{1}{s^2}$

C. $\dfrac{s + e^{-s}}{s^2}$

D. $\dfrac{(s+1)e^{-s}}{s^2}$

E. $\dfrac{1}{s}$

2. Let $y(t)$ be the solution of the initial value problem

$$y'' + 2y' + 3y = \delta(t-1) + u_2(t), \quad y(0) = 0, \quad y'(0) = 1.$$

Compute the Laplace transform of $y(t)$.

A. $Y(s) = \dfrac{1 + e^{-s}}{s^2 + 2s + 3} + \dfrac{e^{-2s}}{s(s^2 + 2s + 3)}$

B. $Y(s) = \dfrac{e^{-s} - 2}{s^2 + 2s} + \dfrac{e^{-2s}}{s(s^2 + 2s)}$

C. $Y(s) = \dfrac{2 + s + e^{-s}}{s^2 + 2s + 3} + \dfrac{e^{-2s}}{s(s^2 + 2s + 3)}$

D. $Y(s) = \dfrac{1 + e^{-2s}}{s^2 + 2s + 3} + \dfrac{e^{-s}}{s(s^2 + 2s + 3)}$

E. $Y(s) = \dfrac{2 - s - e^{-s}}{s^2 + 2s + 3} + \dfrac{e^{-2s}}{s(s^2 + 2s + 3)}$

3. Find the inverse Laplace transform of $F(s) = \dfrac{1 - e^{-\pi s}}{s(s^2+1)}$.

A. $1 - \cos t - u_\pi(t)(1 + \cos t)$

B. $(1 - e^{-\pi t})(1 - \cos t)$

C. $1 - \cos(t) - \delta(t - \pi)$

D. $(1 + e^{\pi t})(1 + \sin(t))$

E. $1 - \cos t - u_\pi(t)(1 - \cos t)$

4. Let $f(t) = \displaystyle\int_0^t (t-\tau)^3 \cos 2\tau \, d\tau$. Find the Laplace transform of $f(t)$.

A. $\dfrac{12}{s^2(s^2+1)}$

B. $\dfrac{6}{s^4} + \dfrac{s}{s^2+4}$

C. $\dfrac{12}{s^4(s^2+4)}$

D. $\dfrac{6}{s^3(s^2+4)}$

E. $\dfrac{6(s-2)}{s^4(s^2+4)}$

5. Find the inverse Laplace transform of $\dfrac{2s+1}{s^2-3s-4}$

A. $(7/5)e^{-4t}+(3/5)e^{t}$

B. $(9/5)e^{4t}+(1/5)e^{-t}$

C. $9e^{4t}+e^{-t}$

D. $2e^{-3t}+e^{t}$

E. $\dfrac{9/5}{t-4}+\dfrac{1/5}{t+1}$

6. Solve the initial value problem

$$y''-4y=\delta(t-1),\quad y(0)=0,\quad y'(0)=0$$

A. $y(t)=u_2(t)\sinh(t-2)$

B. $y(t)=2u_2(t)\sinh(2t-2)$

C. $y(t)=\dfrac{1}{2}u_1(t)\sinh 2t$

D. $y(t)=\dfrac{1}{2}u_1(t)\sinh(2t-1)$

E. $y(t)=\dfrac{1}{2}u_1(t)\sinh(2t-2)$

7. Convert the second order differential equation $y''+5y'+4y=1$ into an equivalent first order system

A. $x_1'=x_2,\quad x_2'=4x_1+5x_2-1$

B. $x_1'=x_2,\quad x_2'=-5x_1-4x_2+1$

C. $x_1=x_2,\quad x_2'=5x_1+4x_2-1$

D. $x_1'=x_2,\quad x_2'=-4x_1-5x_2+1$

E. $x_1'=x_2,\quad x_2'=-4x_1-5x_2$

8. Find the eigenvalues of the matrix $A=\begin{pmatrix}7 & 8\\ -3 & -3\end{pmatrix}$.

A. 3, 1

B. 7, −3.

C. −3, −1

D. −5, 9

E. 2, 4

9. Find the solution of the initial vale problem

$$\mathbf{x}'=\begin{pmatrix}1 & 1\\ 4 & 1\end{pmatrix}\mathbf{x}\quad\text{with}\quad \mathbf{x}(0)=\begin{pmatrix}2\\ 0\end{pmatrix}$$

A. $\begin{pmatrix}1\\ 2\end{pmatrix}e^{3t}+\begin{pmatrix}1\\ -2\end{pmatrix}e^{-t}$

B. $\begin{pmatrix}-2\\ 1\end{pmatrix}e^{3t}+\begin{pmatrix}2\\ 1\end{pmatrix}e^{-t}$

C. $-\frac{1}{2}\begin{pmatrix}-2\\ 1\end{pmatrix}e^{3t}+\frac{1}{2}\begin{pmatrix}1\\ -2\end{pmatrix}e^{-t}$

D. $\begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-t} + \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{3t}$

E. $\frac{1}{2}\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{3t} - \frac{1}{2}\begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-t}$

10. Find the inverse Laplace transform of $Y(s) = \dfrac{(1-3s)e^{-s}}{s^2}$

A. $(t-4)u_1(t)$
B. $(t^2-4)u_3(t)$
C. $tu_1(t)+3$
D. $t^2u_1(t)+e^t$
E. $(e^{3t}+1)u_1(t)$

11. Solve the initial value problem $\begin{cases} y' + y = u_3(t) \\ y(0) = 0 \end{cases}$.

A. $y = 1 - e^{-(t-3)}$
B. $y = u_3(t)\,(1-e^{-t})$
C. $y = u_3(t)\left(1-e^{-(t-3)}\right)$
D. $y = u_3(t) - e^{-(t-3)}$
E. $y = u_3(t)\left(1-e^{-(t-1)}\right)$

12. The Laplace Transform of the solution of the initial value problem $\begin{cases} y'' + 3y' + 4y = u_2(t) \\ y(0) = 1 \\ y'(0) = 3 \end{cases}$ is

A. $\dfrac{e^{-2s}}{s(s^2+3s+4)} + \dfrac{s+6}{s^2+3s+4}$

B. $\dfrac{e^{-2s}}{s(s^2+3s+4)} + \dfrac{s+3}{s^2+3s+4}$

C. $\dfrac{e^{-2s}}{s(s^2+3s+4)} + \dfrac{s}{s^2+3s+4}$

D. $\dfrac{e^{-2s}}{s^2(s^2+3s+4)} + \dfrac{3s+2}{s(s^2+3s+4)}$

E. $\dfrac{3e^{-2s}}{s(s^2+3s+4)^2}$

13. The general solution of the system $\mathbf{x}(t) = \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix} \mathbf{x}(t)$ is

A. $\mathbf{x}(t) = C_1e^{3t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2e^{5t}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

B. $\mathbf{x}(t) = C_1e^{3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2e^{-5t}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

C. $\mathbf{x}(t) = C_1e^{3t}\begin{pmatrix} -1 \\ 1 \end{pmatrix} + C_2e^{5t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

D. $\mathbf{x}(t) = C_1e^{3t}\begin{pmatrix} 4 \\ 1 \end{pmatrix} + C_2e^{5t}\begin{pmatrix} 1 \\ 4 \end{pmatrix}$

E. $\mathbf{x}(t) = C_1e^{3t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2e^{5t}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$