

A. In this course we will use the following definition for the integral:

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(a + i \frac{b-a}{n}\right) \cdot \frac{b-a}{n},$$

and the sum $\sum_{i=1}^n f\left(a + i \frac{b-a}{n}\right) \cdot \frac{b-a}{n}$ is called a *Riemann sum*. (The book gives a more complicated definition which is needed for work with discontinuous functions but will not be needed in this course. You may have seen Riemann sums in your high school course. The Riemann sums we are using have right-hand endpoints and equal subintervals).

Use the above simplified definition of Riemann sums and integrals to:

- (1) Write the following as a definite integral $\int_a^b f(x) dx$ (that is, figure out what a , b , and f are in this example):

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(3 + i \frac{2}{n}\right)^2 \cdot \frac{2}{n}.$$

- (2) Write the following integral as a limit of Riemann sums.

$$\int_1^3 (x^3 - 2x) dx.$$

- B. (1) Evaluate the integral $\int \frac{dx}{3x-2}$.

- (2) Evaluate the integral $\int \frac{e^{2x}}{e^{2x}-2} dx$.

C. Evaluate the following integrals:

(1) $\int \frac{1}{\sqrt{1-9x^2}} dx;$

(2) $\int \frac{1}{1+4x^2} dx;$

(3) $\int \frac{x}{4+x^4} dx;$

(4) $\int \frac{e^x}{1-e^{2x}} dx;$

(5) $\int \frac{e^x}{1-e^x} dx.$

- D. The following problems are a warmup for Chapter 10. DO NOT turn in a printout of your matlab session, just give the answers (to 14 decimal places).

To find matlab on ITaP Windows machines, from the Start menu, go to All Programs, Standard Software, Computational Packages.

- (1) Use the matlab commands

```
> format long
> s=0
> for n=1:10
```

```
s=s+1/4^n;
end
> s
```

to find the sum of the first 10 terms of the series

$$\frac{1}{4^1} + \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^4} + \cdots$$

Then do the same commands but replace

```
> for n=1:10
by
> for n=1:20
```

to find the sum of the first 20 terms.

- (2) Now proceed as in Problem 1 to find the sums of the first 10 and 20 terms of the series

$$\frac{1}{5^1} + \frac{1}{5^2} + \frac{1}{5^3} + \frac{1}{5^4} + \cdots$$

E. (You may turn in printouts for these):

- (1) Use the matlab commands

```
> format long
> s(1:5)=0
> for i=1:5
for n=1:10^i
s(i)=s(i)+1/n^2;
end
end
> s
```

to find the sum of the first 10, 100, 1000, 10000, and 100000 terms of the series

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$

- (2) Use the matlab commands

```
> format long
> s(1:5)=0
> for i=1:5
for n=1:10^i
s(i)=s(i)+1/n;
end
end
> s
```

to find the sum of the first 10, 100, 1000, 10000, and 100000 terms of the series

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$$

- (3) How were your results for Problem 2 different from your results for Problem 1?

F. Use the integral test to decide whether the following series converge or diverge:

(1) $\sum_{n=1}^{\infty} \frac{3n^2}{n^3 + 1}.$

(2) $\sum_{n=1}^{\infty} \frac{3n^2}{(n^3 + 1)^2}.$

(3) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}.$

(4) $\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^2}.$