MA 262
FINAL EXAM INSTRUCTIONS
December 15, 1997

NAME ______________________ INSTRUCTOR ______________________

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. If the cover of your question booklet is WHITE, write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below. If the cover is BLUE, write 02 in the TEST/QUIZ NUMBER boxes and darken the spaces below.

3. On the mark-sense sheet, fill in the instructor's name and the course number.

4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

5. Fill in the SECTION NUMBER boxes with the division and section number of your class. For example, for division 02, section 03, fill in 0203 and blacken the corresponding circles, including the circles for the zeros. (If you do not know your division and section number ask your instructor.)

6. Sign the mark–sense sheet.

7. Fill in your name and your instructor's name on the question sheets above.

8. There are 25 questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work on the question sheets. Turn in both the mark–sense sheets and the question sheets when you are finished.

9. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

10. NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

1. The general solution to $xy' + y = e^{5x}$ is

A. $y = \frac{1}{5}e^{5x} + c$

B. $y = \frac{1}{5x}e^{5x} + cx^{-1}$

C. $y = \frac{1}{6}e^{5x} + ce^{-x}$

D. $y = ce^{5x}$

E. $y = \frac{5}{x}e^{5x} + c$

2. Solutions to $2xy + (x^2 + 1)\frac{dy}{dx} = 0$ satisfy

A. $x^2y + y = c$

B. $x^2y^2 + x = c$

C. $y = e^{\tan^{-1}(x)} + c$

D. $x^2y + 1 = c$

E. $\ln y = -2x\tan^{-1}(x) + c$

3. The substitution $v = y/x$ transforms the equation $\frac{dy}{dx} = \sin(y/x)$ into

A. $v' = \sin(v)$

B. $v' = x\sin(v)$

C. $v' + v = \sin(v)$

D. $xv' + v = \sin(v)$

E. $v' + xv = \sin(v)$

4. The general solution to $y'' = \frac{2}{x}y' + 4x^2$ is

A. $y = \frac{2}{9}x^6 + c_1x^3 + c_2$

B. $y = 2x^2 + c_1x + c_2$

C. $y = x^4 + c_1x + c_2$

D. $y = \frac{1}{3}x^4 + c_1x + c_2$

E. $y = x^4 + c_1x^3 + c_2$

5. A fish tank contains 20 gallons of a salt solution with a concentration of 5 grams of salt per gallon. A salt solution with a concentration of 10 grams/gallon is added to the tank at a rate of 2 gallons per minute. At the same time, water is drained from the tank at a rate of 2 gallons per minute. How many grams of salt are in the tank after 10 minutes?

A. $200e^{-1} + 100$

B. $200 - 100e^{-1}$

C. $200e - 100$

D. 100

E. 200

6. Which of the following sets forms a basis for $\mathbb{R}^3$?
(i) $\{(2, -1, 0), (1, -2, 0), (1, -3, 1)\}$
(ii) $\{(\frac{1}{2}, 1, 1), (0, 3, 0), (0, 0, 2), (1, 1, 0)\}$
(iii) $\{(1, 1, 1), (0, \frac{1}{2}, \frac{1}{2}), (2, 0, 0)\}$
(iv) $\{(\frac{1}{2}, 1, 1), (0, 1, 0), (1, 2, 0)\}$

A. (i) only

B. (i), (ii) and (iv)

C. (i), (iii) and (iv)

D. (i) and (iv)

E. None of the above.

7. Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be defined by $T(\mathbf{x}) = A\mathbf{x}$ where $A = \begin{bmatrix} 1 & 1 & -1 & -3 \\ 0 & 1 & 1 & -4 \\ 2 & 2 & -2 & -6 \end{bmatrix}$. Then the dimension of $\mathrm{Ker}\,(T)$ is

A. 0

B. 1

C. 2

D. 3

E. 4

8. Let $\mathcal{S}$ denote the subspace of $\mathbb{R}^3$ equal to the set of all solutions to $A\vec{x} = 0$ where

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ -1 & 0 & 2 \end{bmatrix}.$$

Which of the following vectors forms a basis for $\mathcal{S}$?

A. $(-2, 1, 1)$

B. $(2, 1, 0)$

C. $(2, -1, 1)$

D. $(-1, 0, 1)$

E. None of the above.

9. If $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation satisfying

$$T(1,0,0) = (0,0,-1), \quad T(0,1,0) = (1,1,0), \quad T(0,0,1) = (1,1,2),$$

then $T(1,2,3) =$

A. $(5, 5, 5)$

B. $(2, 1, 0)$

C. $(2, -1, 1)$

D. $(0, 0, 0)$

E. $(1, 2, 3)$

10. For which value(s) of k are the vectors $(2, -k, 0)$, $(1, 2, 2)$, and $(0, 1, -k)$ linearly *dependent*?

A. No values of k.

B. $k = -2$

C. $k \neq -2$

D. $k = 0$

E. All values of k.

11. For which value(s) of k are the vectors $(2, -k, 0)$ and $(1, 2, 2)$ linearly *dependent*?

A. No values of k.

B. $k = -2$

C. $k \neq -2$

D. $k = 0$

E. All values of k.

12. For which value(s) of k are the vectors $(2, -k, 0)$, $(1, 2, 2)$, $(0, 1, -k)$, and $(0, 1, k)$ linearly *dependent*?

A. No values of k.

B. $k = -2$

C. $k \neq -2$

D. $k = 0$

E. All values of k.

13. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{bmatrix}$, and $B = A^{-1}$, then the entry b_{23} of B is

A. 0

B. -4

C. 4

D. $1/6$

E. $-2/3$

14. Determine all values of k so that the following system has **no solution**.

$$\begin{aligned} x_1 + x_2 + 3x_3 &= 1 \\ 2x_1 + 4x_2 + 5x_3 &= 1 \\ x_1 - x_2 + k^2 x_3 &= -k \end{aligned}$$

A. $k \neq \pm 2$

B. $k = \pm 2$

C. $k = 2$

D. $k = -2$

E. $-2 < k < 2$

15. If one solution of $y'' + y' - 2y = f(x)$ is $y(x) = \ln x$, then the general solution is

A. $c_1 \ln x$

B. $c_1 \ln x + c_2 e^x + c_3 e^{-2x}$

C. $c_1 e^x + c_2 e^{-2x} + \ln x$

D. $c_1 e^x + c_2 e^{-2x}$

E. $c_1 e^{-x} + c_2 e^{2x} + \ln x$

16. If $y(x)$ is the solution of $y'' - y' - 2y = 0$ satisfying $y(0) = 1$ and $y'(0) = -1$, then $y(1) =$

A. $e^{-1} + 2e^2$

B. e^{-1}

C. e^2

D. $e^2 - e^{-1}$

E. $2e^2$

17. The general solution of the equation $y'' + 2y' + 5y = 0$ is

A. $c_1 e^{-x} \cos 2x + c_2 e^{-x} \sin 2x$

B. $c_1 e^{(-1+i)x} + c_2 e^{(-1-i)x}$

C. $c_1 e^{-x} + c_2 x e^{-x}$

D. $c_1 e^{2x} \cos x + c_2\, i\, e^{2x} \sin x$

E. $c_1 e^{x} + c_2 e^{-3x}$

18. The general solution of $D(D^2 + 1)(D + 1)^2 y = 0$ is

A. $c_1 x + c_2 \cos x + c_3 \sin x + e^{-x}(c_4 + c_5 x)$

B. $c_1 + c_2 e^{x} + e^{-x}(c_3 + c_4 x + c_5 x^2)$

C. $c_1 \cos x + c_2 \sin x + e^{-x}(c_3 + c_4 x)$

D. $c_1 \cos x + c_2 \sin x + c_3 e^{-x} + c_4$

E. $c_1 \cos x + c_2 \sin x + e^{-x}(c_3 + c_4 x) + c_5$

19. In the method of undetermined coefficients, the appropriate trial solution for $(D^2 + 2D + 2)(D + 1)y = \sin x + xe^{-x}$ is

A. $A \sin x + Bxe^{-x}$

B. $A \sin x + B \cos x + Cxe^{-x}$

C. $A \sin x + B \cos x + (C + Dx)e^{-x}$

D. $A \sin x + B \cos x + x(C + Dx)e^{-x}$

E. $A \sin x + Bx^2e^{-x}$

20. A particular solution for $y'' + 2y' + y = x^{-1}e^{-x}$ is

A. $(\ln x - x^2)e^{-x}$

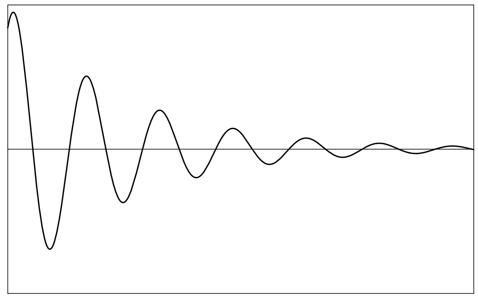
B. $Ax^{-1}e^{-x}$

C. Axe^{-x}

D. $\ln x \, e^{-x}$

E. $x \ln x \, e^{-x}$

21. For which values of the parameter α will the equation $y'' + \alpha y' + y = 0$ have solutions whose graphs are similar to the following graph?



A. $\alpha < 2$

B. $\alpha = 0$

C. $0 < \alpha < 2$

D. $\alpha > 2$

E. all α

22. All the solutions of a 2×2 homogeneous linear system of differential equations $\vec{x}\,' = A\vec{x}$ tend to zero as $t \to \infty$ if the eigenvalues of A are equal to:

A. 1 and -1

B. i and $-i$

C. $1 + i$ and $1 - i$

D. $-1 + i$ and $-1 - i$

E. None of the above.

23. The general solution of the linear system of differential equations

$$\begin{aligned} x_1' &= x_1 + 2x_2 \\ x_2' &= 4x_1 + 3x_2 \end{aligned}$$

is equal to

A. $c_1 \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix} + c_2 \begin{bmatrix} e^{5t} \\ 2e^{5t} \end{bmatrix}$

B. $c_1 \begin{bmatrix} e^{-t} \\ e^{-t} \end{bmatrix} + c_2 \begin{bmatrix} e^{5t} \\ -2e^{5t} \end{bmatrix}$

C. $c_1 \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix} + c_2 \begin{bmatrix} e^{5t} \\ -2e^{5t} \end{bmatrix}$

D. $c_1 \begin{bmatrix} e^{-t} \\ e^{-t} \end{bmatrix} + c_2 \begin{bmatrix} e^{5t} \\ 2e^{5t} \end{bmatrix}$

E. None of the above.

24. The function $x_2(t)$ determined by the initial value problem

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= -x_1 \end{aligned}$$

with initial conditions $x_1(0) = 1$ and $x_2(0) = 1$ is given by

A. $x_2 = -\sin t + \cos t$

B. $x_2 = \sin t + \cos t$

C. $x_2 = \frac{1}{2}(e^t + e^{-t})$

D. $x_2 = \cos t$

E. $x_2 = ie^{it} - ie^{-it}$

25. The 2×2 matrix $A = \begin{bmatrix} -3 & -1 \\ 2 & -1 \end{bmatrix}$ has complex eigenvalues $r = -2 \pm i$. An eigenvector corresponding to $r = -2 + i$ is $\begin{pmatrix} 1 \\ -1 - i \end{pmatrix}$. The system

$$\vec{x}\,' = A\vec{x} + \begin{pmatrix} 3 \\ -4 \end{pmatrix} e^{-2t}$$

has one solution given by $\vec{x}(t) = \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$. What is the general solution to the system?

A. $c_1 \begin{pmatrix} 1 \\ -1 - i \end{pmatrix} e^{(-2+i)t} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{2t}$

B. $c_1 \begin{pmatrix} \cos t \\ \sin t - \cos t \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} \sin t \\ -\sin t - \cos t \end{pmatrix} e^{-2t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

C. $c_1 \begin{pmatrix} \cos t \\ -\cos t \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 0 \\ -\sin t \end{pmatrix} e^{-2t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

D. $c_1 \begin{pmatrix} 1 \\ -1 - i \end{pmatrix} e^{(-2+i)t} + c_2 \begin{pmatrix} -1 \\ 1 + i \end{pmatrix} e^{(-2-i)t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

E. $c_1 \begin{pmatrix} \cos t \\ -\cos t \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 0 \\ -\sin t \end{pmatrix} e^{-2t} + c_3 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$