1. A cylindrical tank with radius 2 in and height 12 in is being filled with water at a rate of 8 in^3/min. How fast is the height of the water increasing when $h < 12$ in? *Hint:* $V = \pi r^2 h$

A. 32π in/min

B. 6π in/min

C. 48π in/min

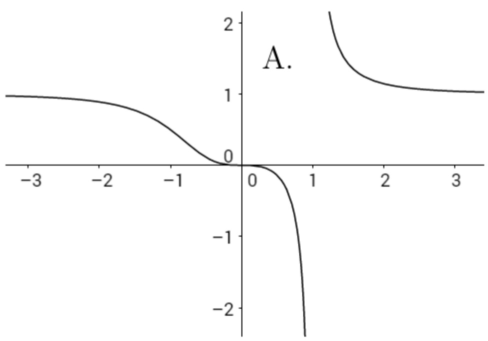
D. $\boxed{\frac{2}{\pi} \text{ in/min}}$

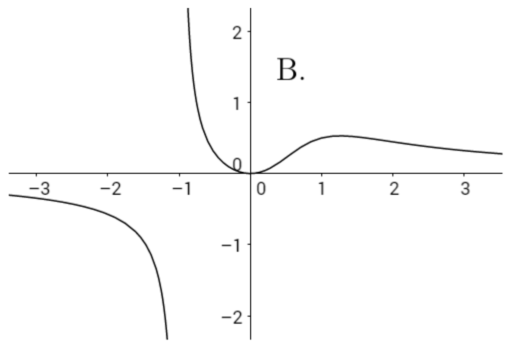
E. $\frac{1}{6\pi}$ in/min

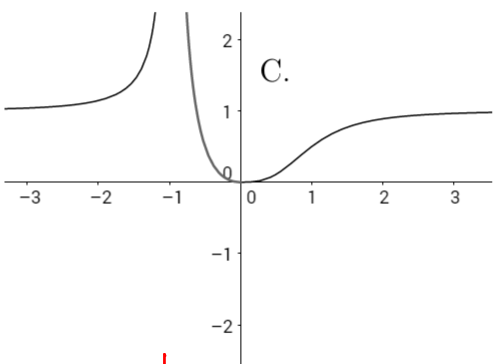
$$8 = \frac{dV}{dt} = \pi \cdot 2^2 \frac{dh}{dt}$$

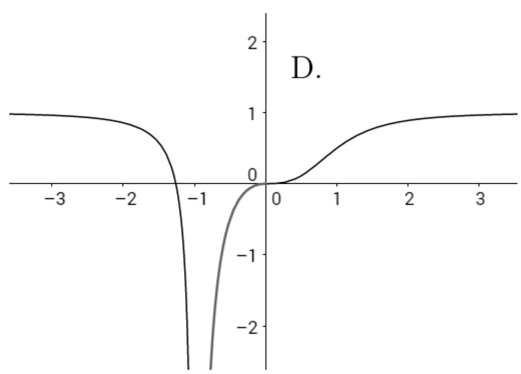
$$\frac{dh}{dt} = \frac{8}{\pi \cdot 2^2} = \frac{2}{\pi}$$

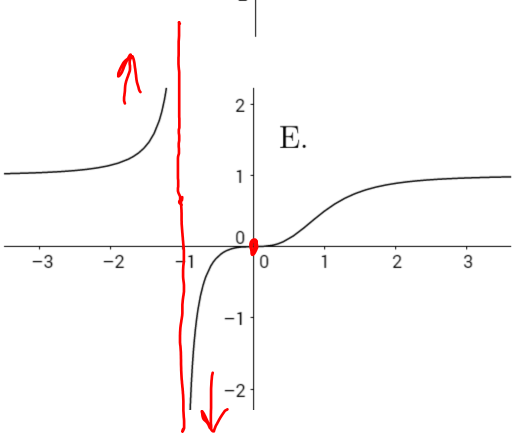
2. Which of the following is the graph of $y = \dfrac{x^3}{x^3+1}$?











− − − − − 0 + + + + + → Sign x^3
0

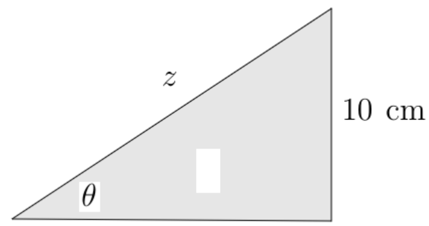
− − − 0 + + + + + + + + → Sign x^3+1
−1

Boom!

+ + + + − − − − − 0 + + + + + → Sign $\dfrac{x^3}{x^3+1}$
−1 0

3. Suppose z denotes the length of the hypotenuse of a right triangle, and that θ is an acute angle in the triangle whose opposite side has a fixed length of 10 cm. If $\theta = \frac{\pi}{6}$, then $z = 20$ cm. Use differentials to find dz, the approximate change in z, if $d\theta = -0.05$ radians.

$$\sin\theta = \frac{10}{z}$$



$$z = \frac{10}{\sin\theta}$$

A. 1/10 cm

B. $\boxed{\sqrt{3} \text{ cm}}$

C. $\sqrt{3}/2$ cm

D. 1/2 cm

E. 1/3 cm

$$dz = \frac{dz}{d\theta} \cdot d\theta = \left(\frac{-10}{(\sin\theta)^2} \cdot \cos\theta \right) d\theta$$

$$dz = \left(\frac{-10}{(1/2)^2} \cdot \frac{\sqrt{3}}{2} \right)(-0.05)$$

$$= \sqrt{3}\,(20) \cdot \underbrace{(.05)}_{1/20} = \sqrt{3}$$

4. Suppose $f(x) = \sqrt{3}\sin x + \cos x$. If M is the absolute maximum of f on the interval $[0, \pi]$, and m is the absolute minimum value on the same interval, what is the sum $M + m$?

A. 0

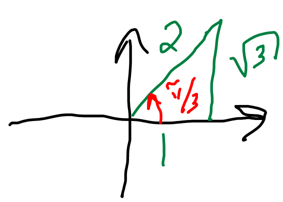
B. $\sqrt{3} - 1$

C. $\boxed{1}$

D. 2

E. 3

$$f'(x) = \sqrt{3}\cos x - \sin x = 0 \qquad \frac{\sin x}{\cos x} = \sqrt{3}$$



$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \qquad \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\tan x = \sqrt{3}$$

$$\boxed{x = \frac{\pi}{3}}$$

Crt. #

x	$f(x) = \sqrt{3}\sin x + \cos x$
0	$\sqrt{3} \cdot 0 + 1 = 1$
$\frac{\pi}{3}$	$\sqrt{3} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} = 2 \leftarrow M$
π	$\sqrt{3} \cdot 0 - 1 = -1 \leftarrow m$

$$M + m = 1$$

5. Which of these statements describes the graph of $y = e^{2x} + e^{-x}$?

A. One local maximum and no inflection points

B. $\boxed{\text{One local minimum and no inflection points}}$

C. One local minimum and one inflection point

D. One local maximum and one inflection point

E. None of the above

$y' = 2e^{2x} - e^{-x} = 0$

$2e^{2x} = e^{-x}$

$2 = e^{-3x}$

$\ln 2 = -3x$

$\boxed{x = -\frac{1}{3}\ln 2}$

crt. #

$y'' = 4e^{2x} + e^{-x}$ always > 0

No inflection pts.

$y''(-\frac{1}{3}\ln 2) > 0$: 2nd Der. Test $\Rightarrow$ local min

6. At noon, ship A is 20 miles west of ship B. Ship A is sailing north at 6 miles/hr and ship B is sailing east at 4 miles/hr. How fast is the distance between the ships changing at 5:00 PM?

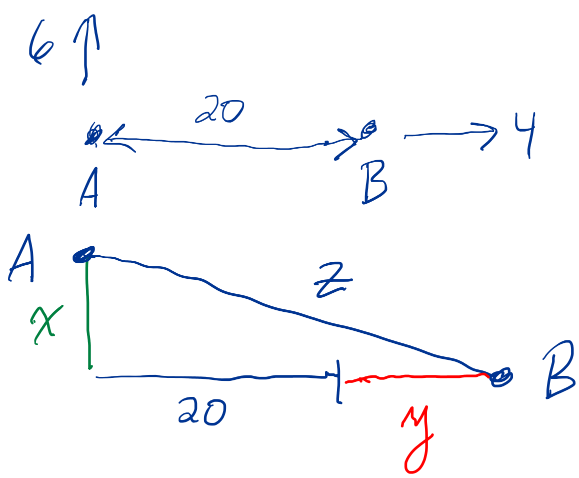
A. 10 miles/hr

B. $2\sqrt{13}$ miles/hr

C. $\boxed{\frac{34}{5} \text{ miles/hr}}$

D. $\frac{132}{\sqrt{30}}$ miles/hr

E. 5 miles/hr



$\frac{dx}{dt} = 6$

$\frac{dy}{dt} = 4$

$x^2 + (20+y)^2 = z^2$

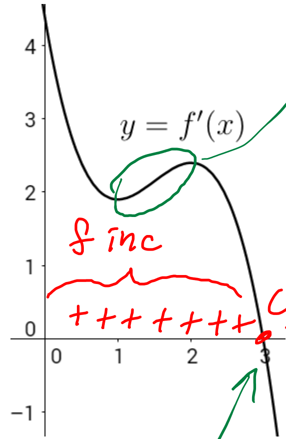
$2x\frac{dx}{dt} + 2(20+y)\frac{dy}{dt} = 2z\frac{dz}{dt}$

$x = 30,\ y = 20,\ z = 50$ at 5:00

At 5:00 : $\frac{dz}{dt} = \frac{x\frac{dx}{dt} + (y+20)\frac{dy}{dt}}{z} = \frac{30\cdot 6 + (20+20)\cdot 4}{50}$

$= \frac{180+160}{50} = \frac{340}{50}$ mph

7. The graph of f', the **derivative of** f, is pictured below:



derivative of f' positive So $f'' > 0$ here Concave up

f inc

$+ + + + + + + \; 0 \; - - -$ Sign f'

$x = 3$ a local max by 1st Der Test

Which of the following statements are true?

I. f has a local minimum at $x = 3$.

✓ II. f is concave up on the interval $(1, 2)$.

✓ III. f is increasing on the interval $(0, 3)$.

A. Only one of the statements is true.

B. I and II

C. $\boxed{\text{II and III}}$

D. I and III

E. All three statements are true.

8. $\displaystyle\lim_{x\to 0}\frac{1-\cos 4x}{x^2} \overset{L'H}{=} \frac{0}{0}$

$$= \lim_{x\to 0}\frac{-(-\sin 4x)\cdot 4}{2x} \overset{L'H}{=} \frac{0}{0}$$

$$= \lim_{x\to 0}\frac{(\cos 4x)\cdot 4\cdot 4}{2} = \frac{(\cos 0)\cdot 16}{2} = 8$$

A. 0

B. ∞

C. 4

D. $\boxed{8}$

E. 16

9. $\lim_{x\to 0^+} (1+\sin(x))^{1/x} =$

A. 1

B. $\ln 2$

C. 0

D. ∞

E. $\boxed{e}$

$$y = (1+\sin x)^{1/x}$$

$$\ln y = \frac{1}{x}\cdot \ln(1+\sin x)$$

$$\lim_{x\to 0+} \ln y = \lim_{x\to 0+} \frac{\ln(1+\sin x)}{x} \overset{L'H}{=} \frac{0}{0}$$

$$= \lim_{x\to 0+} \frac{\frac{1}{1+\sin x}\cdot \cos x}{1} = \frac{\frac{1}{1+0}\cdot 1}{1} = 1$$

So $y = e^{\ln y} \longrightarrow e^{1}$ as $x \to 0^+$

10. If you use the linear approximation of $f(x) = x^{100}$ at $a = 100$ to find an approximate value of 99^{100}, the approximate value found is

A. $\boxed{0}$

B. 100^{99}

C. 99^{99}

D. $100^{100} - 99$

E. $100^{100} - 100$

$$f(x) \approx f(a) + f'(a)(x-a)$$

$$\begin{cases} f(x) = x^{100} \\ f'(x) = 100x^{99} \end{cases}$$

$$x^{100} \approx 100^{100} + 100\cdot 100^{99}(x-100)$$

let $x = 99$

So $\begin{cases} f(100) = 100^{100} \\ f'(100) = 100\cdot 100^{99} \end{cases}$

$$99^{100} \approx 100^{100} + 100\cdot 100^{99}(99-100)$$

$$= 100^{100} + 100^{100}(-1) = 0$$

11. If $f(5) = 6$ and the derivative of f is always less than or equal to 10, what is the largest value $f(10)$ could take?

A. 50
B. $\boxed{56}$
C. 16
D. 44
E. 66

$$\frac{f(10) - f(5)}{10 - 5} = f'(c) \quad c \in (5, 10), \text{ MVT}$$

$$\frac{f(10) - 6}{5} \leq 10$$

$$f(10) - 6 \leq 50$$

$$f(10) \leq 56$$

12. The minimum value of $\underbrace{x^3 - 3x + 9}_{f(x)}$ on the interval $[-3, 2]$ is

A. $\boxed{-9}$
B. 7
C. 11
D. -1
E. 3

$$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 0$$

$x = \pm 1$

crt. #s

	x	$x^3 - 3x + 9$	
end	-3	$-27 + 9 + 9 = -9$	min.
crt. #'s	-1	$-1 + 3 + 9 = 11$	max
crt. #'s	1	$1 - 3 + 9 = 7$	
end	2	$8 - 6 + 9 = 11$	max